

RADICALI E POTENZE FRAZIONARIE

$$a^b$$

↑ base ← esponente

a^m con m intero positivo : $a^m = \underbrace{a \cdot a \cdot \dots \cdot a}_{m \text{ volte}} \quad a \in \mathbb{R}$

Principali proprietà

$$a^m \cdot a^n = a^{m+n}$$

$$\underbrace{a \cdot \dots \cdot a}_m \cdot \underbrace{a \cdot \dots \cdot a}_n \rightsquigarrow a \text{ compare } m+n \text{ volte}$$

$$(a^m)^n = a^{m \cdot n}$$

Esempio $(a^2)^3 = (a \cdot a)^3 = \underbrace{(a \cdot a)(a \cdot a)(a \cdot a)}_{\text{in tutto 6 volte}}$

$$(ab)^m = a^m \cdot b^m$$

Esempio $(a \cdot b)^3 = (a \cdot b) \cdot (a \cdot b) \cdot (a \cdot b) = a^3 \cdot b^3$

$$(a \pm b)^m = \text{utente di farlo}$$

Esponente intero negativo: $a^{-5} = \frac{1}{a^5} \quad a \neq 0$

In questo modo continua a essere vero che $a^{m+n} = a^m \cdot a^n$ anche se m ed n non sono più positivi

Esempi $a^2 \cdot a^{-5} = \cancel{a} \cdot \cancel{a} \frac{1}{\cancel{a} \cdot \cancel{a} \cdot a \cdot a \cdot a} = a^{-3} = a^{2-5}$

Esponente razionale: $a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad a > 0$

Esponente reale: diventa più difficile da definire, ma restano vere sempre le stesse proprietà

$$a^{x+y} = a^x \cdot a^y \quad a > 0 \quad x \in \mathbb{R} \quad y \in \mathbb{R}$$

$$(a^x)^y = a^{xy} \quad a > 0 \quad " \quad "$$

$$a^x \cdot b^x = (a \cdot b)^x \quad a > 0 \quad b > 0 \quad x \in \mathbb{R}$$

Da queste si deduce anche $a^{x-y} = a^x \cdot a^{-y} = a^x \cdot \frac{1}{a^y} = \frac{a^x}{a^y}$

Occhio! $2^{2^3} = 2^8 \quad (2^2)^3 = 2^6 \quad \text{per convenzione}$

Esercizi ① $\sqrt{a} = 2$
 $a = 4$

$$\sqrt[9]{9} = 3$$

 $a = 2$

$$a^{\frac{1}{2}} = 2 \rightsquigarrow \text{elevo al } \square: (a^{\frac{1}{2}})^2 = 2^2 \rightsquigarrow a = 4$$

$$9^{\frac{1}{a}} = 3 \rightsquigarrow (3^2)^{\frac{1}{a}} = 3 \rightsquigarrow 3^{\frac{2}{a}} = 3 \rightsquigarrow \frac{2}{a} = 1 \rightsquigarrow a = 2$$

② $\sqrt[9]{16} = 2$
 $a = 4$

$$\sqrt[9]{16} = 4 \quad 4^{\frac{2}{a}} = 4$$

 $a = 2$

③ $\sqrt[4]{a} = 3$
 $a = 81$

$$a^{\frac{1}{2}} = 5$$

 $a = 25$

$$a^{\frac{1}{4}} = 3 \rightsquigarrow \text{elevo alla quarta}$$

$$(a^{\frac{1}{2}})^2 = 5^2 \rightsquigarrow a = 5^2$$

$$\textcircled{4} \quad a^{3/2} = 27$$

$$a = 9$$

$$a^{-1/2} = \frac{1}{4}$$

$$a = 16$$

$$a^{3/2} = 3^3 \rightsquigarrow \text{elevo alla } \frac{2}{3} : a = (3^3)^{2/3} = 3^2 = 9$$

$$a^{-1/2} = \frac{1}{a^{1/2}} = \frac{1}{\sqrt{a}} = \frac{1}{4} \rightsquigarrow \sqrt{a} = 4 \rightsquigarrow a = 16$$

$$\textcircled{5} \quad 2^{-a} = \frac{1}{8} \quad a = 3$$

$$8^a = 4 \quad a = \frac{2}{3}$$

$$2^{-a} = \frac{1}{2^a} = \frac{1}{8} = \frac{1}{2^3}$$

$$(2^3)^a = 2^2 \rightsquigarrow 2^{\boxed{3a}} = 2^{\boxed{2}}$$

$$\textcircled{6} \quad \sqrt[a]{16} = 8 \quad a = \frac{4}{3}$$

$$2^{20} - 2^{19} = 2^a \quad a = 19$$

$$(2^4)^{1/a} = 2^3 \rightsquigarrow 2^{4/a} = 2^3$$

$$2^{20} = 2^{1+19} = 2^1 \cdot 2^{19} = 2 \cdot 2^{19}$$

$$\rightsquigarrow \frac{4}{a} = 3 \rightsquigarrow a = \frac{4}{3}$$

$$\textcircled{7} \quad \sqrt{2} \cdot \sqrt{2} = \sqrt{a} \quad a = 4$$

$$\sqrt{2} \cdot \sqrt{5} = \sqrt{a} \quad a = 10$$

$$2^{\boxed{1/2}} \cdot 2^{\boxed{1/2}} = (2 \cdot 2)^{\boxed{1/2}}$$

$$2^{1/2} \cdot 5^{1/2} = (2 \cdot 5)^{1/2}$$

$$\textcircled{8} \quad \sqrt{2} \cdot \sqrt{3} = \sqrt[a]{6} \quad a = 2$$

$$\sqrt{2} \cdot \sqrt{3} = \sqrt[4]{a} \quad a = 36$$

$$\sqrt{2} \cdot \sqrt{2} = \sqrt{6} = \sqrt[4]{a}$$

$$\rightsquigarrow a^{1/4} = 6^{1/2} \rightsquigarrow \text{elevo alla } 4$$

$$\textcircled{9} \quad \sqrt{\sqrt{2}} = \sqrt[a]{2} \quad a = 4$$

$$\sqrt[3]{2} = \sqrt[a]{2} \quad a = 6$$

$$\sqrt{\sqrt{2}} = \sqrt{2^{1/2}} = (2^{1/2})^{1/2} = 2^{1/4}$$

$$(2^{1/3})^{1/2} = 2^{1/3 \cdot 1/2} = 2^{1/6}$$

$$\textcircled{10} \quad \sqrt[3]{\sqrt{27}} = \sqrt{a} \quad a=3 \qquad \sqrt{\sqrt{2}} = \sqrt[8]{a} \quad a=4$$

$$\left[(3^3)^{\frac{1}{2}} \right]^{\frac{1}{3}} = (3^3)^{\frac{1}{2} \cdot \frac{1}{3}} = 3^{3 \cdot \frac{1}{2} \cdot \frac{1}{3}} = 3^{\frac{1}{2}} = \sqrt{3}$$

$$2^{\frac{1}{4}} = a^{\frac{1}{8}} \rightsquigarrow \text{elevo alla 8: } a = 2^2 = 4$$

$$\textcircled{11} \quad \sqrt{2\sqrt{2}} = \sqrt[a]{8} \quad a=4 \qquad \sqrt{4\sqrt{2}} = \sqrt{2^a} \quad a = \frac{5}{2}$$

$$2\sqrt{2} = 2^1 \cdot 2^{\frac{1}{2}} = 2^{1+\frac{1}{2}} = 2^{\frac{3}{2}} = (2^3)^{\frac{1}{2}} = \sqrt{8}$$

$$\sqrt{2\sqrt{2}} = \sqrt{\sqrt{8}} = \sqrt[4]{8}$$

$$\text{Alternativa: } 2\sqrt{2} = \sqrt{4} \cdot \sqrt{2} = \sqrt{8}$$

$$4\sqrt{2} = \sqrt{16} \cdot \sqrt{2} = \sqrt{32} = 2^{\frac{5}{2}} \rightsquigarrow \sqrt{4\sqrt{2}} = 2^{\frac{5}{4}} = 2^{\frac{5}{2}}$$

$$\textcircled{12} \quad \sqrt{2} \cdot \sqrt[3]{2} \cdot \sqrt[4]{2} = 2^a$$

$$2^{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} = 2^a \rightsquigarrow \text{trovo } a$$

$$\sqrt[6]{8} = \sqrt[a]{4}$$

$$2^{\frac{3}{6}} = 2^{\frac{2}{a}} \rightsquigarrow a=4$$

$$\textcircled{13} \quad \sqrt[a]{2} \cdot \sqrt[3]{2} = 2$$

$$2^{\frac{1}{a} + \frac{1}{3}} = 2^1$$

$$\frac{1}{a} + \frac{1}{3} = 1 \rightsquigarrow \text{trovo } a$$

$$\sqrt[3]{2} : \sqrt[a]{a} = 2^{-1} \quad a=256$$

$$\frac{2^{\frac{1}{3}}}{a^{\frac{1}{6}}} = \frac{1}{2}$$

$$2^{\frac{1}{3}+1} = a^{\frac{1}{6}}$$

$$2^{\frac{4}{3}} = a^{\frac{1}{6}} \rightsquigarrow \text{elevo alla 6}$$

$$2^8 = a$$

$$\textcircled{14} \quad \sqrt{3\sqrt{2\sqrt{3}}} = \sqrt[a]{a} \quad a=9\sqrt{2}$$

$$2\sqrt{3} = \sqrt{4} \cdot \sqrt{3} = \sqrt{12}$$

$$3\sqrt[4]{12} = \sqrt[4]{81} \cdot \sqrt[4]{12} = \sqrt[4]{972}$$

$$\sqrt{2} \cdot \sqrt[a]{4} = 2$$

$$2^{\frac{1}{2}} \cdot 2^{\frac{2}{a}} = 2^1$$

$$\frac{1}{2} + \frac{2}{a} = 1 \Rightarrow a = 4$$

$$\textcircled{15} \quad \sqrt{2} + \sqrt{2} = a\sqrt{2}$$

$$a = 2$$

$$\sqrt{2} + \sqrt{2} = \sqrt{a} \quad a = 8$$

$$\sqrt{2} + \sqrt{2} = 2\sqrt{2} = \sqrt{4} \cdot \sqrt{2} = \sqrt{8}$$

$$\textcircled{16} \quad \sqrt{8} \cdot \sqrt[3]{2^a} \cdot \sqrt[4]{8} = 8$$

$$2^{\frac{3}{2}} \cdot 2^{\frac{a}{3}} \cdot 2^{\frac{3}{4}} = 2^3$$

$$\frac{3}{2} + \frac{a}{3} + \frac{3}{4} = 3 \Rightarrow \text{trovo } a$$

$$\sqrt{32} - \sqrt{2} = \sqrt{a} \quad a = 18$$

$$\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$$

$$4\sqrt{2} - \sqrt{2} = 3\sqrt{2} = \sqrt{9} \cdot \sqrt{2} = \sqrt{18}$$

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