

DISEQUAZIONI

Obiettivo generale: data una funzione $f(x)$, suddividere $\mathbb{R}$ in quattro sottoinsiemi disgiunti

→ gli x per cui $f(x) = 0$

→ " $f(x) > 0$

→ " $f(x) < 0$

→ gli x per cui non ha senso calcolare $f(x)$ per colpa di problemi burocratici (tipo denominatori nulli)

Primo grado

ACHTUNG! Non incasinarsi con i segni

$$\textcircled{1} \quad 3x \geq 2 \quad \rightsquigarrow \quad x \geq \frac{2}{3}$$

$$3x \geq -2 \quad \rightsquigarrow \quad x \geq -\frac{2}{3}$$

$$-3x \geq 2 \quad \rightsquigarrow \quad x \leq -\frac{2}{3}$$

$$-3x \geq -2 \quad \rightsquigarrow \quad x \leq \frac{2}{3}$$

$$-2 \geq 3x \quad \rightsquigarrow \quad x \leq -\frac{2}{3}$$

Quando moltiplico o divido per una negativa devo girare i segni

$$\textcircled{2} \quad 2(x+1) + x \leq 3x+2$$

$$2x+2+x \leq 3x+2$$

$$3x+2 \leq 3x+2$$

Questa è sempre verificata

$$0 \leq 0 \quad \text{☺}$$

Occhio! $7 \geq 5$ e $7 \geq 7$ sono vere

$$2(x+1) + x < 3x+2 \quad \rightsquigarrow \quad 3x+2 < 3x+2 \quad \rightsquigarrow \quad 0 < 0 \quad \text{?}$$

Mai verificata

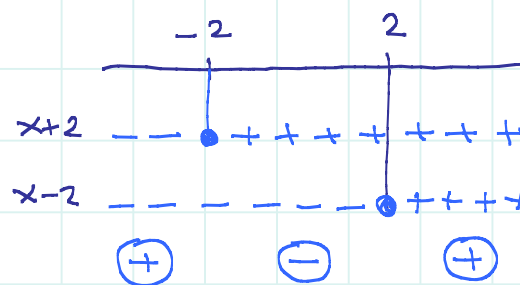
$$-3x-1 \leq -7+x \quad \rightsquigarrow \quad 4x \geq 6 \quad \rightsquigarrow \quad x \geq \frac{3}{2}$$

Secondo grado

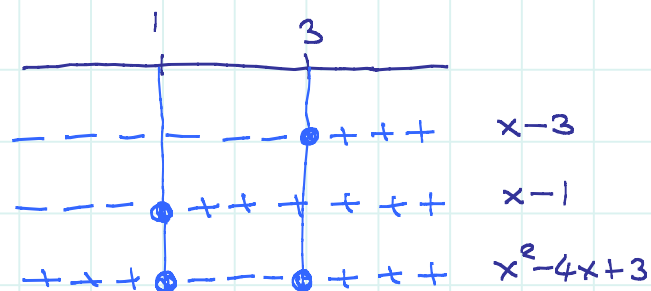
① $x^2 \leq 4 \quad \leadsto \quad x \leq \pm 2 \quad \text{NO, PLEASE}$
 $\leadsto -2 \leq x \leq 2 \quad \text{oppure} \quad x \in [-2, 2]$

Si pensa da $x^2 - 4 \leq 0 \leadsto$ radici $x=2$ e $x=-2$
 $\leadsto$ valori interni

Perché? $(x+2)(x-2) \leq 0$



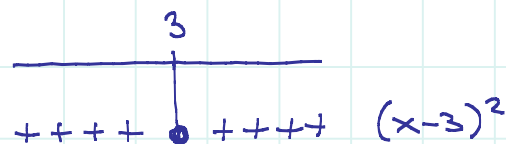
② $x^2 - 4x + 3$ Studio il segno
 $(x-3)(x-1)$



③ $-x^2 \geq x \quad \text{Nemmeno pensare di dividere}$
 $x^2 + x \leq 0 \quad \leadsto \quad x(x+1) \leq 0 \quad \leadsto$ radici $x=-1$ e $x=0$
 $\leadsto$ valori interni $x \in [-1, 0]$

④ $x^2 - 6x + 9$

$(x-3)^2$



⑤ $-4x^2 - 4x - 1 \geq 0 \quad \leadsto \quad 4x^2 + 4x + 1 \leq 0 \quad \leadsto \quad (2x+1)^2 \leq 0$
 $\leadsto \quad x = -\frac{1}{2} \quad x \in \{-\frac{1}{2}\}$

Quando il discriminante è negativo non ci sono annullamenti, quindi è sempre + o sempre - a seconda del coeff. di x^2

$$\textcircled{6} \quad -x^2 + 3x - 2025 \leq 0$$

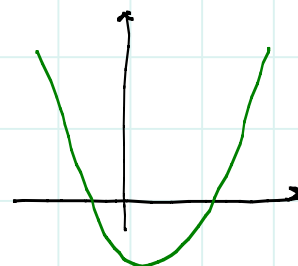
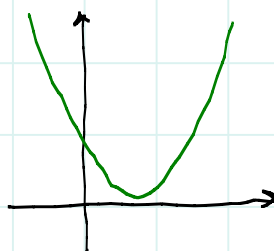
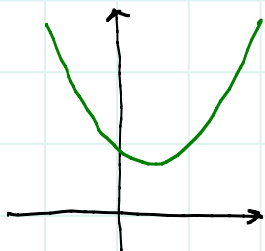
$$x = \frac{-3 \pm \sqrt{9 - 8100}}{-2}$$

nessuna radice reale $\leadsto$ sempre < 0 $\leadsto$ sempre verificata

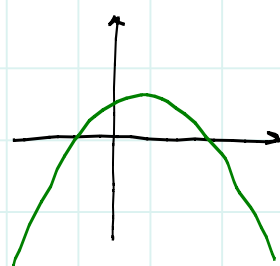
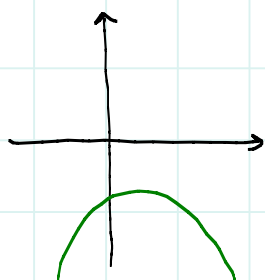
Interpretazione grafica

$$ax^2 + bx + c = 0$$

$$a > 0$$



$$a < 0$$



$$b^2 - 4ac < 0$$

$$b^2 - 4ac = 0$$

$$b^2 - 4ac > 0$$

$$\textcircled{7} \quad 2x^2 \geq 1 \quad \leadsto \quad 2x^2 - 1 \geq 0 \quad \leadsto \quad \text{radici: } x = \pm \frac{1}{\sqrt{2}}$$

$$(\sqrt{2}x + 1)(\sqrt{2}x - 1) \geq 0$$

$$\text{Soluzione: } (-\infty, -\frac{1}{\sqrt{2}}] \cup [\frac{1}{\sqrt{2}}, +\infty)$$

$$\textcircled{8} \quad (x+2)^2 > x \quad \leadsto \quad x^2 + 4x + 4 > x \quad \leadsto \quad x^2 + 3x + 4 > 0$$

$$\text{Radici: } x = \frac{-3 \pm \sqrt{9 - 16}}{2}$$

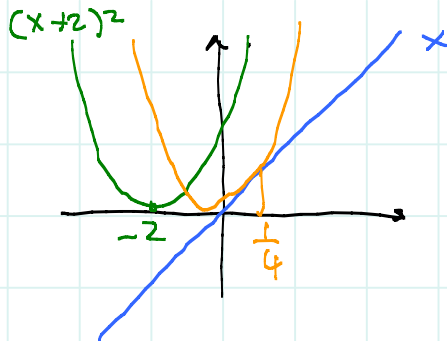
$\Delta < 0 \leadsto$ sempre > 0
 $\leadsto$ sempre verificata

$$(x + \frac{1}{4})^2 > x \quad \leadsto \quad x^2 + \frac{x}{2} + \frac{1}{16} > x \quad \leadsto \quad x^2 - \frac{x}{2} + \frac{1}{16} > 0$$

$$\leadsto 16x^2 - 8x + 1 > 0$$

$$(4x - 1)^2 > 0$$

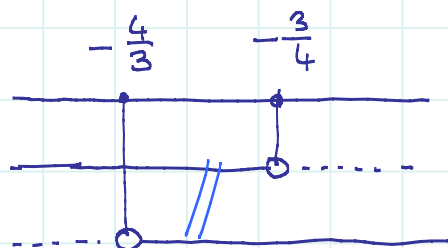
$$\text{Soluzione: } x \neq \frac{1}{4}$$



SISTEMI DI DISEQUAZIONI

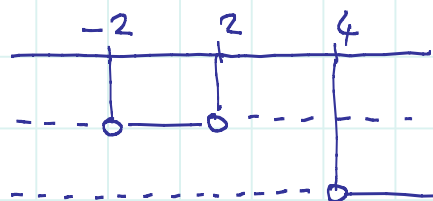
Devono essere tutte verificate contemporaneamente

$$\textcircled{1} \begin{cases} -4x > 3 \\ -3x < 4 \end{cases} \quad \begin{cases} x < -\frac{3}{4} \\ x > -\frac{4}{3} \end{cases}$$



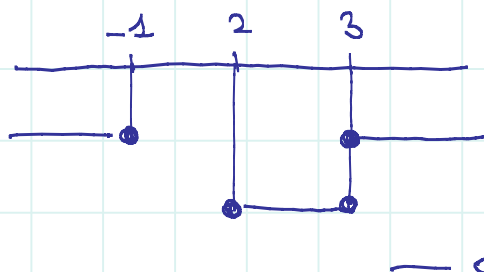
Soluzione del sistema: $x \in \left(-\frac{4}{3}, -\frac{3}{4}\right)$ $-\frac{4}{3} < x < -\frac{3}{4}$

$$\textcircled{2} \begin{cases} x^2 - 4 < 0 \\ x - 4 > 0 \end{cases} \quad \begin{cases} x \in (-2, 2) \\ x > 4 \end{cases}$$



Nessuna soluzione

$$\textcircled{3} \begin{cases} x^2 - 2x \geq 3 \\ x^2 + 6 \leq 5x \end{cases} \quad \begin{cases} x^2 - 2x - 3 \geq 0 \\ x^2 - 5x + 6 \leq 0 \end{cases} \quad \begin{cases} (x-3)(x+1) \geq 0 \\ (x-2)(x-3) \leq 0 \end{cases}$$



Soluzione: solo $x = 3$
 $\{3\}$