

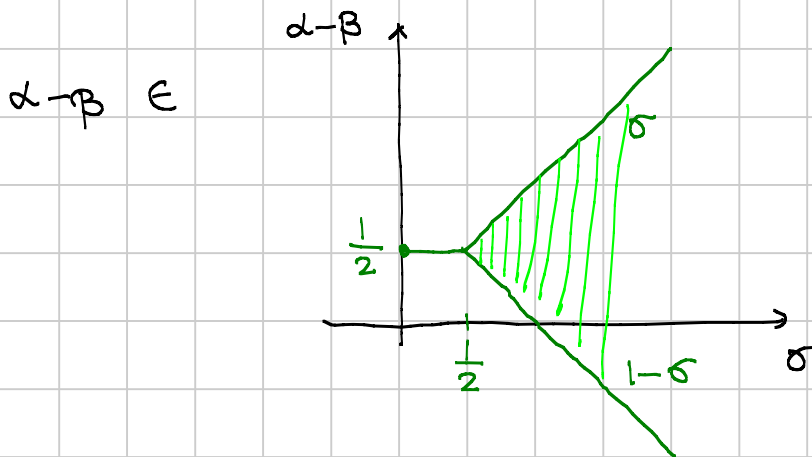
$$\ddot{u} + 2\delta A^\sigma \dot{u} + Au = 0 \quad \delta > 0, \sigma \geq 0$$

As always we reduce ourselves to

$$\ddot{u}_k + 2\delta \lambda_k^{2\sigma} \dot{u}_k + \lambda_k^2 u_k = 0$$

Result

① Well-posedness in $D(A^\alpha) \times D(A^\beta)$ provided that



- if $\sigma \in [0, \frac{1}{2}]$, then $\alpha - \beta = \frac{1}{2}$ (standard hyperb. gap)
- if $\sigma > \frac{1}{2}$, then $1 - \sigma \leq \alpha - \beta \leq \sigma$.

Remark Assume $\sigma \leq \frac{1}{2}$, assume $(u_0, u_1) \in D(A^{3/4}) \times D(A^{1/4})$
Then for every $t > 0$ it turns out that

$$(u(t), \dot{u}(t)) \in D(A^{1/4 + \frac{1}{2}}) \times D(A^{1/4})$$

Assume $\sigma = 4$ and (u_0, u_1) as before. then

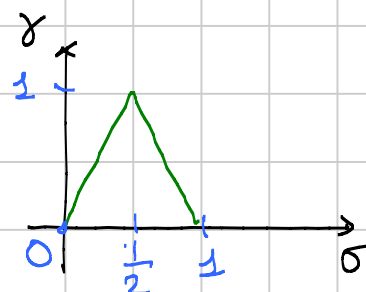
$$(u(t), \dot{u}(t)) \in D(A^{18}) \times D(A^{1/4})$$

② $u^{(m)} \in \mathcal{D}(A^{\beta - (m-1)\sigma})$ for every $m \geq 1$ if $\sigma \geq \frac{1}{2}$
 $\uparrow$
 m derivatives w.r.t time

③ Assume that $\sigma \in (0, 1)$ then for every $t > 0$

$$(u(t), \dot{u}(t)) \in G_{\varphi, \alpha, \beta} \times G_{\varphi, \alpha, \beta}$$

with $\varphi(x) = x^\gamma$ with



④ Assume that $\sigma > 1$

Then $u^{(m)} \in \mathcal{D}(A^{\alpha + m(\sigma-1)})$

$\forall t > 0$

higher derivatives are in better spaces

"Proof" (sketch of)

$$\ddot{u}_k + 2\delta \lambda_k^{2\sigma} \dot{u}_k + \lambda_k^2 u_k = 0$$

Char equ : $x^2 + 2\delta \lambda_k^{2\sigma} x + \lambda_k^2 = 0$

$$x_{1,2} := -\delta \lambda_k^{2\sigma} \pm \sqrt{\delta^2 \lambda_k^{4\sigma} - \lambda_k^2}$$

Case $\sigma > \frac{1}{2}$ or $\sigma = \frac{1}{2}$ and $\delta > 1 \leadsto 2$ real solutions (diff.)

$$x_1 \sim -2\delta \lambda_k^{2\sigma}$$

$$x_2 = \frac{\lambda_k^2}{x_1} \sim -\frac{1}{2\delta} \lambda_k^{2-2\sigma}$$

$u_k(t)$ is the sum of four terms...

$$u_k(t) = c_1 e^{x_1 t} + c_2 e^{x_2 t}$$

→ compute c_1 and c_2 in terms of u_{0k} and u_{1k} and obtain 4 terms.

$$\begin{aligned} & \pm u_{0k} \frac{x_2}{x_1 - x_2} e^{+x_1 t} \\ & \sim u_{0k} \frac{\lambda_k^{2-2\sigma}}{\lambda_k^{2\sigma}} e^{-\lambda_k^{2\sigma} t} \\ & \downarrow \alpha \\ & \underbrace{\lambda_k^{2-4\sigma}}_{\text{good}} \end{aligned}$$

$$\begin{aligned} & \pm u_{0k} \frac{x_1}{x_1 - x_2} e^{+x_2 t} \\ & \sim u_{0k} \frac{\lambda_k^{2\sigma}}{\lambda_k^{2\sigma}} e^{-\lambda_k^{2-2\sigma} t} \\ & \downarrow \alpha \end{aligned}$$

$$u_{0k} \frac{x_1 x_2}{x_1 - x_2} e^{\dots} \sim u_{0k} \frac{\lambda_k^{2\sigma} \lambda_k^{2-2\sigma}}{\lambda_k^{2\sigma} \lambda_k^{2-2\sigma}}$$

$\downarrow \alpha$

exactly the same

$$\begin{aligned} & \pm u_{1k} \frac{1}{x_1 - x_2} e^{x_1 t} \\ & \sim u_{1k} \frac{1}{\lambda_k^{2\sigma}} e^{-\lambda_k^{2\sigma} t} \\ & \downarrow \beta \\ & \frac{1}{\lambda_k^{2\sigma}} \end{aligned}$$

$$u_{1k} \frac{x_1}{x_1 - x_2} \sim u_{1k} \frac{\lambda_k^{2\sigma}}{\lambda_k^{2\sigma}}$$

$\downarrow \beta$

$$\begin{aligned} & \pm u_{1k} \frac{1}{x_1 - x_2} e^{x_2 t} \\ & \sim u_{1k} \frac{1}{\lambda_k^{2\sigma}} e^{-\lambda_k^{2-2\sigma} t} \\ & \downarrow \beta \\ & \frac{1}{\lambda_k^{2\sigma}} \end{aligned}$$

$$u_{1k} \frac{x_2}{x_1 - x_2} \sim u_{1k} \frac{\lambda_k^{2-2\sigma}}{\lambda_k^{2\sigma}}$$

$\downarrow \beta$

$\frac{1}{\lambda_k^{4\sigma-2}}$

The space where u lies is the worst between

$$\alpha \text{ and } \beta + \sigma \quad \leadsto \quad \beta + \sigma \geq \alpha \quad \leadsto \quad \boxed{\alpha - \beta \leq \sigma}$$

The space where i dies is the worst between

$$\alpha + \sigma - 1$$

$$\beta$$

$$\underbrace{\beta + 2\sigma - 1}_{\text{too good}}$$

$$\leadsto \beta \leq \alpha + \sigma - 1 \quad \leadsto \boxed{\alpha - \beta \geq 1 - \sigma}$$