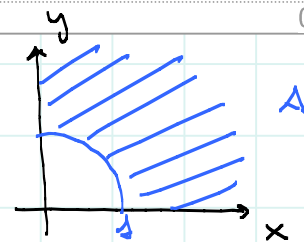


Esempio 1 $\iint_A \frac{x}{x^2+y^6} dx dy$



1° modo Paragghiamo gli esponenti

$$x = u^3$$

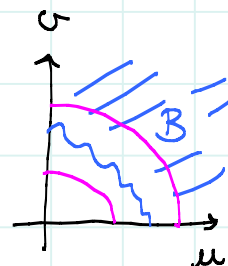
$$y = v$$

$$\iint_A \frac{x}{x^2+y^6} dx dy = \iint_B \frac{u^3}{u^6+v^6} \cdot 3u^2 du dv$$

$$J = \begin{pmatrix} 3u^2 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A = x \geq 0, y \geq 0, x^2 + y^2 \geq 1$$

$$B = u \geq 0, v \geq 0, u^6 + v^6 \geq 1$$



Come è fatto B non importa perché

$$3 \iint_B \frac{u^5}{u^6+v^6} du dv \text{ conv. } \Leftrightarrow \iint_{u^2+v^2 \geq 1} \frac{u^5}{u^6+v^6} du dv$$

Brutal mode: $\frac{\rho^5}{\rho^6} = \frac{1}{\rho}$ e quindi diverge

$$\frac{u^5}{u^6+v^6} = \frac{\rho^5 \cos^5 \theta}{\rho^6 (\cos^6 \theta + \sin^6 \theta)} \geq \frac{1}{\rho} \frac{\cos^5 \theta}{2}$$

$$\iint_{\substack{u^2+v^2 \geq 1 \\ u \geq 0 \\ v \geq 0}} \frac{u^5}{u^6+v^6} du dv \geq \frac{1}{2} \int_1^{+\infty} d\rho \int_0^{\frac{\pi}{4}} d\theta \underbrace{\frac{1}{\rho} \cos^5 \theta}_{\text{numero} > 0} = +\infty$$

Alternativa

$$\begin{aligned} \iint_{\substack{u^2+v^2 \geq 1 \\ u \geq 0 \\ v \geq 0}} \dots du dv &\geq \iint_{\substack{u^2+v^2 \geq 1 \\ 0 \leq v \leq u}} \dots \\ &= \int_1^{+\infty} d\rho \int_0^{\frac{\pi}{4}} d\theta \frac{1}{2\rho} \boxed{\cos^5 \theta} \rho \\ &\geq \left(\frac{1}{\sqrt{2}}\right)^5 \end{aligned}$$

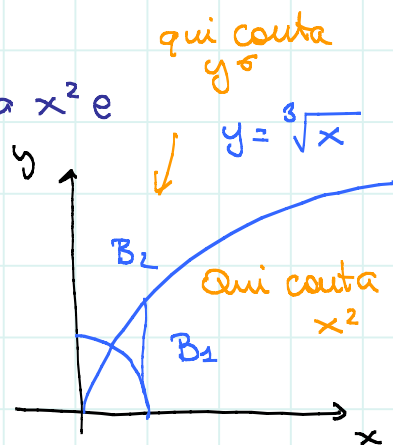


2° modo

$$\frac{x}{x^2+y^6}$$

Cerco di capire, al denominatore, quando comanda x^2 e quando comanda y^6

$$x^2 \geq y^6 \Leftrightarrow y \leq \sqrt[3]{x}$$



$$\iint_B \dots = \iint_{B_1} \dots + \iint_{B_2} \dots$$

$$\begin{aligned} \iint_{B_1} \frac{x}{x^2+y^6} dx dy &\geq \iint_{B_1} \frac{x}{2x^2} dx dy \\ &\geq \int_1^{+\infty} dx \int_0^{\sqrt[3]{x}} dy \frac{1}{2x} \\ &= \frac{1}{2} \int_1^{+\infty} \frac{\sqrt[3]{x}}{x} dx = \text{diverge} \end{aligned}$$

↗
conteniamo

Esempio 2

$$\iint_Q \frac{\arctan(xy)}{x^4+y^4} dx dy$$

↑
primo quadrante

Due problemi: → nell'origine
→ all'infinito

$$\sim \frac{xy}{x^4+y^4} \sim \frac{1}{\rho^2} \rightsquigarrow \text{diverge}$$

$$\sim \frac{\pi}{2} \frac{1}{\rho^4} \rightsquigarrow \text{converge}$$

$$\iint_{\substack{x^2+y^2 \geq 1 \\ x \geq 0 \\ y \geq 0}} \frac{\arctan(xy)}{x^4+y^4} dx dy \leq \iint \dots \frac{\pi}{2} \frac{1}{x^4+y^4} dx dy$$

... polari ... converge

Con il problema a 0 abbiamo

$$\begin{aligned} &\int_0^1 dp \int_0^{\frac{\pi}{2}} d\theta \frac{\arctan(\rho^2 \cos \theta \sin \theta)}{\rho^4 (\cos^4 \theta + \sin^4 \theta)} \geq \frac{1}{2} \int_0^1 dp \frac{1}{\rho^3} \int_0^{\frac{\pi}{2}} \arctan(\dots) d\theta \\ &\geq \frac{1}{2} \int_0^1 dp \frac{1}{\rho^3} \int_{\theta_1}^{\theta_2} \arctan(\rho^2 \cos \theta \sin \theta) d\theta \end{aligned}$$

↑
param

$0 < \theta_1 < \theta_2 < \frac{\pi}{2}$

$$\geq \frac{1}{2} \int_0^1 d\varrho \frac{1}{\varrho^3} \int_{\theta_1}^{\theta_2} \arctan(m\varrho^2) d\theta$$

$\uparrow$
 $m = \min\{\cos\theta \sin\theta : \theta \in [\theta_1, \theta_2]\} > 0$

$$= \frac{1}{2} (\theta_2 - \theta_1) \int_0^1 \frac{\arctan(m\varrho^2)}{\varrho^3} d\varrho$$

diverge per confronto
 asintotico con $\frac{1}{\varrho}$

$$\left[\lim_{\varrho \rightarrow 0^+} \frac{\arctan(m\varrho^2)}{\varrho^3} \cdot \varrho = \lim_{\varrho \rightarrow 0^+} \frac{\arctan(m\varrho^2)}{\varrho^2} = m \neq 0 \neq +\infty \right]$$

— 0 — 0 —

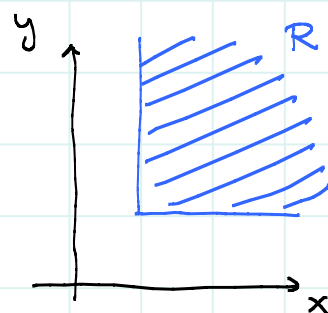
Esempio 3 $\iint_Q \frac{1}{1+x^2y^2} dx dy$

Brutal mode: solo problema all'infinito, $\varphi \sim \frac{1}{\varrho^4} \Rightarrow$ converge

NO! NO!

$$R = [1, +\infty) \times [1, +\infty)$$

$$\iint_R \frac{1}{1+x^2y^2} dx dy \text{ converge}$$



$$\iint_R \frac{1}{1+x^2y^2} dx dy \leq \iint_R \frac{1}{x^2y^2} dx dy$$

$$= \int_1^{+\infty} dx \int_1^{+\infty} dy \frac{1}{x^2y^2} = \underbrace{\int_1^{+\infty} \frac{1}{x^2} dx}_{\text{conv.}} \underbrace{\int_1^{+\infty} \frac{1}{y^2} dy}_{\text{conv.}} < +\infty$$

Definisco

$$C := \{(x, y) \in \mathbb{R}^2 : x \geq 1, 0 \leq y \leq \frac{1}{x}\}$$

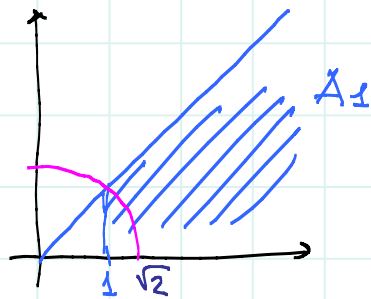
$$\iint_Q \frac{1}{1+x^2y^2} dx dy \geq \iint_C \frac{1}{1+x^2y^2} dx dy$$

$$xy \leq 1 \text{ in } C \rightarrow \geq \frac{1}{2} \iint_C 1 dx dy = \frac{1}{2} \int_1^{+\infty} \frac{1}{x} dx = +\infty$$



Esempio 4 $\iint_{A_a} \frac{1}{x^2+y^2} dx dy$

dove $A_a := \{(x,y) \in \mathbb{R}^2 : x \geq 1, 0 \leq y \leq x^a\}$
 $\uparrow$
 $a > 0$



$$\begin{aligned} \iint_{A_1} \frac{1}{x^2+y^2} dx dy &\geq \int_{\frac{1}{\sqrt{2}}}^{+\infty} dp \int_0^{\frac{\pi}{4}} d\theta \frac{1}{p^2} \cdot p \\ &= \frac{\pi}{4} \int_{\frac{1}{\sqrt{2}}}^{+\infty} \frac{1}{p} dp = \text{diverge} \end{aligned}$$

Osservo che $A_1 \subseteq A_a$ per ogni $a \geq 1$, quindi allo stesso modo

$$\iint_{A_a} \dots = +\infty \text{ per ogni } a \geq 1$$

Congetture che per $a < 1$ converge

$$\iint_{A_a} \dots = \int_1^{+\infty} dx \int_0^{x^a} dy \frac{1}{x^2+y^2}$$

$$\leq \int_1^{+\infty} dx \int_0^{x^a} dy \left(\frac{1}{x^2} \right)$$

$$= \int_1^{+\infty} \frac{x^a}{x^2} dx = \int_1^{+\infty} \frac{1}{x^{2-a}} dx \quad \text{e questo converge se } 2-a > 1, \text{ cioè } a < 1$$



— o — o —