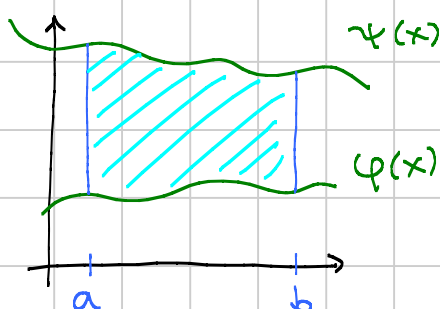


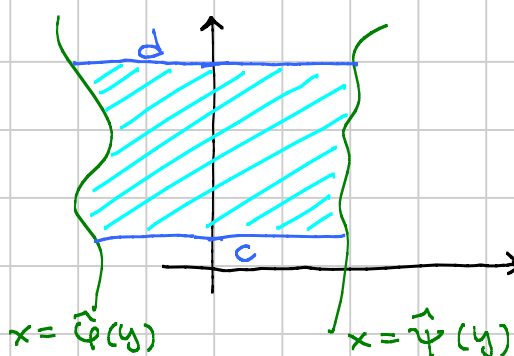
Insieme normali nel piano (Lezioni 23/24 di AM2-14)

$$A = \{ (x, y) : a \leq x \leq b, \varphi(x) \leq y \leq \psi(x) \} \quad \text{Normale asse } x$$

$$B = \{ (x, y) : c \leq y \leq d, \hat{\varphi}(y) \leq x \leq \hat{\psi}(y) \} \quad \text{Normale asse } y$$

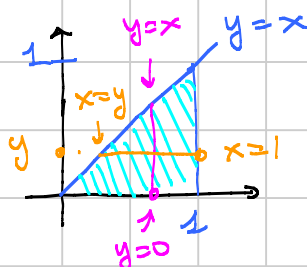


Asse x



Asse y

Esempio 1



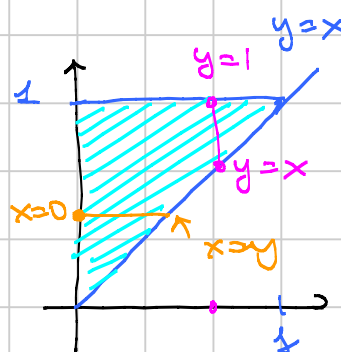
Asse x : $A = \{ (x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq x \}$

Asse y : $A = \{ (x, y) \in \mathbb{R}^2 : 0 \leq y \leq 1, y \leq x \leq 1 \}$

Esempio 2

$$A = \{ (x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, x \leq y \leq 1 \}$$

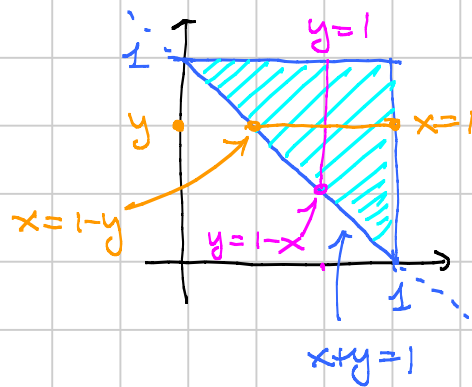
$$= \{ \quad \quad : 0 \leq y \leq 1, 0 \leq x \leq y \}$$



Esempio 3

$$A = \{(x,y) \in \mathbb{R}^2: 0 \leq x \leq 1, 1-x \leq y \leq 1\}$$

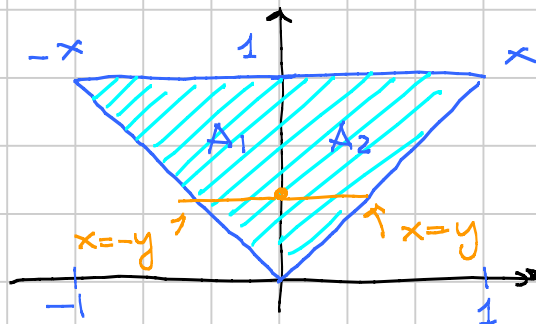
$$= \{(x,y) \in \mathbb{R}^2: 0 \leq y \leq 1, 1-y \leq x \leq 1\}$$



Esempio 4

$$A = \{(x,y) \in \mathbb{R}^2: -1 \leq x \leq 1, |x| \leq y \leq 1\}$$

conetta, ma $|x|$ può essere fastidioso
su un integrale



$$= \{(x,y) \in \mathbb{R}^2: -1 \leq x \leq 0, -x \leq y \leq 1\} \\ \cup \{(x,y) \in \mathbb{R}^2: 0 \leq x \leq 1, x \leq y \leq 1\}$$

(A1)

(A2)

Normale rispetto all'asse y si può fare in un solo pezzo senza usare valori assoluti

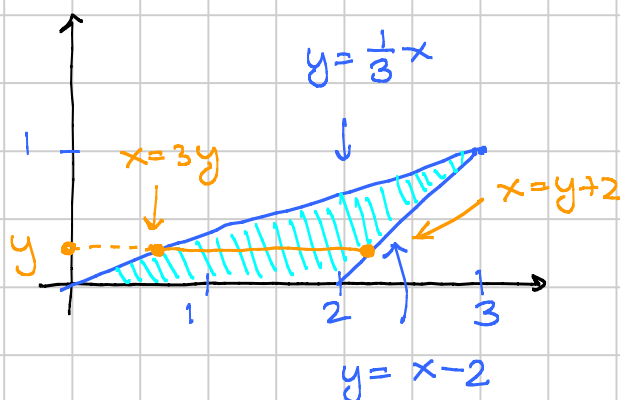
$$A = \{(x,y) \in \mathbb{R}^2: 0 \leq y \leq 1, -y \leq x \leq y\}$$

Esempio 5

Normale risp. asse x servono 2 pezzi

$$\{(x,y) \in \mathbb{R}^2: 0 \leq x \leq 2, 0 \leq y \leq \frac{x}{3}\}$$

$$\cup \{(x,y) \in \mathbb{R}^2: 2 \leq x \leq 3, x-2 \leq y \leq \frac{x}{3}\}$$



Normale risp. asse y basta un pezzo solo

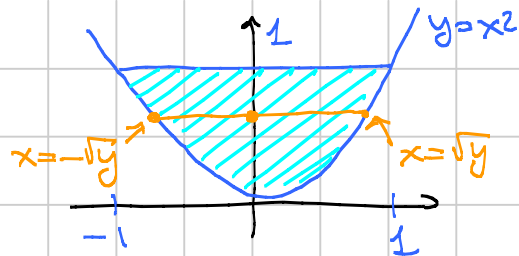
$$\{(x,y) \in \mathbb{R}^2: 0 \leq y \leq 1, 3y \leq x \leq y+2\}$$

[Verificare per qualche valore di y]

Esempio 6

$$A = \{(x, y) \in \mathbb{R}^2: -1 \leq x \leq 1, x^2 \leq y \leq 1\}$$

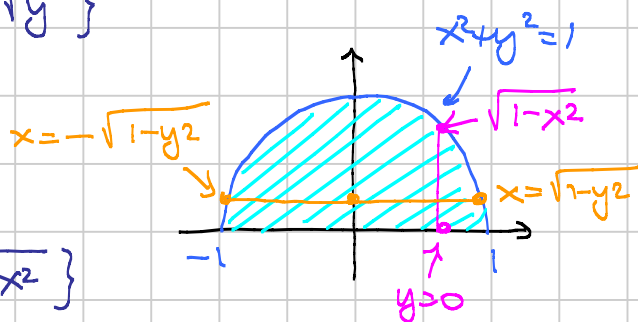
$$= \{(x, y) \in \mathbb{R}^2: 0 \leq y \leq 1, -\sqrt{y} \leq x \leq \sqrt{y}\}$$



Esempio 7

$$A = \{(x, y) \in \mathbb{R}^2: -1 \leq x \leq 1, 0 \leq y \leq \sqrt{1-x^2}\}$$

$$= \{(x, y) \in \mathbb{R}^2: 0 \leq y \leq 1, -\sqrt{1-y^2} \leq x \leq \sqrt{1-y^2}\}$$

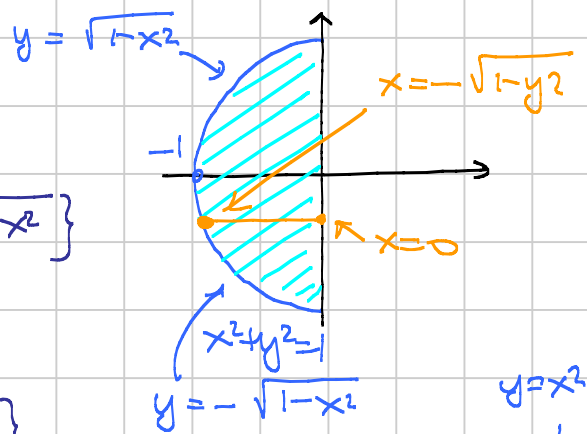


Esempio 8

$$A = \{(x, y) \in \mathbb{R}^2: -1 \leq x \leq 0, -\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}\}$$

(asse x)

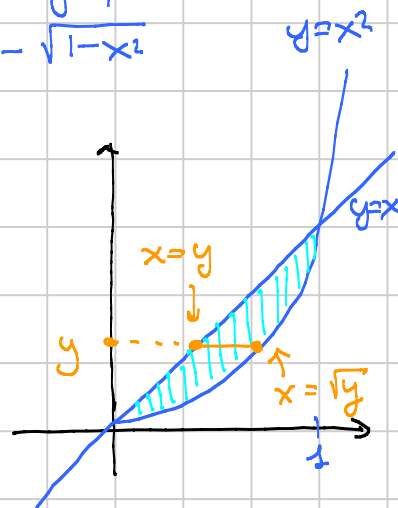
$$= \{(x, y) \in \mathbb{R}^2: -1 \leq y \leq 1, -\sqrt{1-y^2} \leq x \leq 0\}$$



Esempio 9

$$A = \{(x, y) \in \mathbb{R}^2: 0 \leq x \leq 1, \underset{\substack{\uparrow \\ \text{parab}}}{x^2} \leq y \leq \underset{\substack{\uparrow \\ \text{retta}}}{x}\}$$

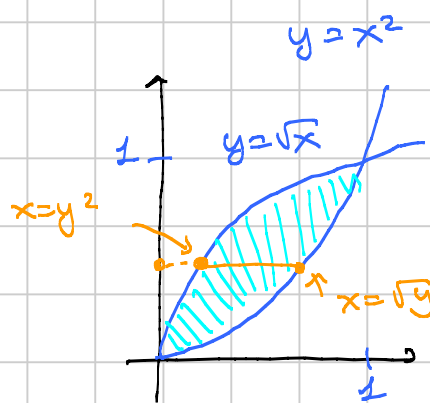
$$= \{(x, y) \in \mathbb{R}^2: 0 \leq y \leq 1, y \leq x \leq \sqrt{y}\}$$



Esempio 10

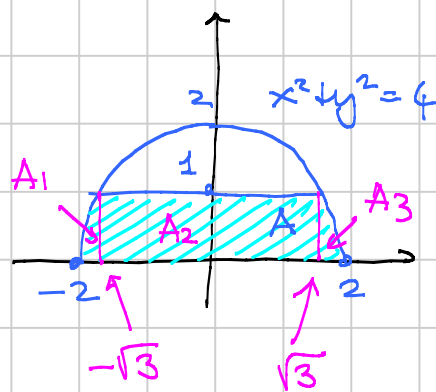
$$A = \{(x, y) \in \mathbb{R}^2: 0 \leq x \leq 1, x^2 \leq y \leq \sqrt{x}\}$$

$$= \{(x, y) \in \mathbb{R}^2: 0 \leq y \leq 1, y^2 \leq x \leq \sqrt{y}\}$$



Esempio 11

Per scriverlo come normale rispetto all'asse x occorre dividere in 3 pezzi e devo anche calcolare le ascisse dei p.t. di intersezione



$$A_1 = \{(x, y) \in \mathbb{R}^2: -2 \leq x \leq -\sqrt{3}, 0 \leq y \leq \sqrt{4-x^2}\}$$

↑ parte alta circonf.

$$A_2 = \{(x, y) \in \mathbb{R}^2: -\sqrt{3} \leq x \leq \sqrt{3}, 0 \leq y \leq 1\} = [-\sqrt{3}, \sqrt{3}] \times [0, 1]$$

rettangolo

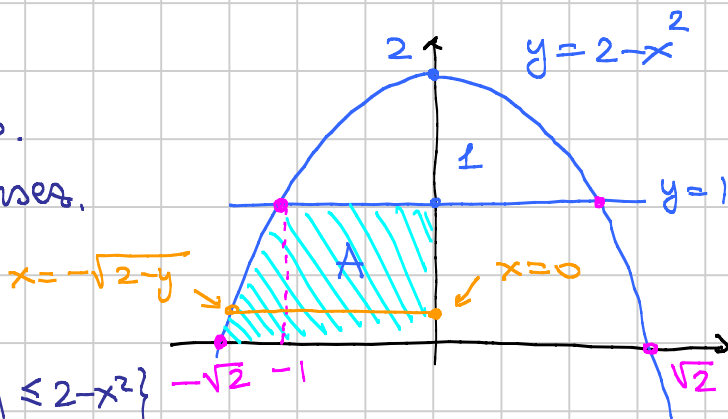
$$A_3 = \{(x, y) \in \mathbb{R}^2: \sqrt{3} \leq x \leq 2, 0 \leq y \leq \sqrt{4-x^2}\}$$

Come insieme normale rispetto all'asse y basta un pezzo solo

$$A = \{(x, y) \in \mathbb{R}^2: 0 \leq y \leq 1, -\sqrt{1-y^2} \leq x \leq \sqrt{1-y^2}\}$$

Esempio 12

Per scriverlo come normale risp. asse x occorre calcolare le intersez. e usare 2 pezzi



$$\{(x, y) \in \mathbb{R}^2: -\sqrt{2} \leq x \leq -1, 0 \leq y \leq 2-x^2\}$$

$$\cup \{(x, y) \in \mathbb{R}^2: -1 \leq x \leq 0, 0 \leq y \leq 1\}$$

↑ rettangolo $[-1, 0] \times [0, 1]$

Normale rispetto all'asse y basta un pezzo solo

$$\{(x, y) \in \mathbb{R}^2: 0 \leq y \leq 1, -\sqrt{2-y} \leq x \leq 0\}$$

Esempio 13

Basta un pezzo solo in entrambi i versi

$$A = \{ (x, y) \in \mathbb{R}^2 : -1 \leq x \leq 0,$$

$$-\sqrt{4-4x^2} \leq y \leq 0 \} \quad (\text{asse } x)$$

$$= \{ (x, y) \in \mathbb{R}^2 : -2 \leq y \leq 0,$$

$$-\frac{\sqrt{4-y^2}}{2} \leq x \leq 0 \}$$

— o — o —

