

DISEGNO DI INSIEMI DEL PIANO

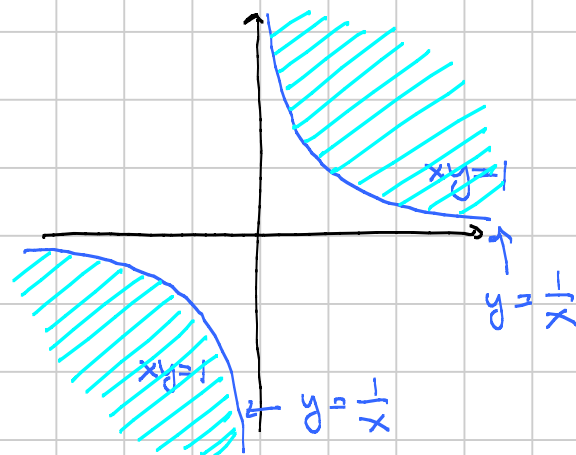
$$\textcircled{1} \{(x, y) \in \mathbb{R}^2 : xy \geq 1\}$$

$$xy = 1 \Leftrightarrow y = \frac{1}{x}$$

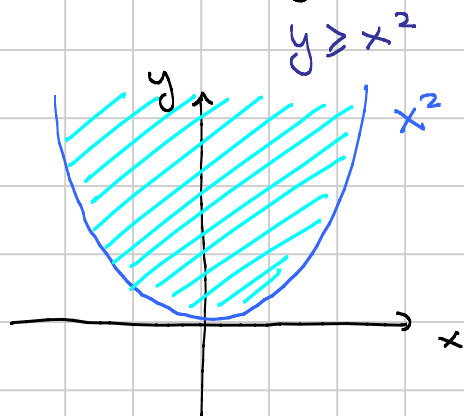
$$xy \geq 1 \Leftrightarrow y \geq \frac{1}{x} \quad \text{NO!!!}$$

Se divido, conservo i versi delle diseq.
Solo se sto dividendo per roba > 0 ,
altrimenti devo invertire i versi !!

$$xy \geq 1 \Rightarrow \begin{cases} \text{se } x > 0 \text{ divido } \leadsto y \geq \frac{1}{x} \\ \text{se } x < 0 \text{ divido e } \boxed{\text{inverto}} \leadsto y \boxed{\leq} \frac{1}{x} \end{cases}$$

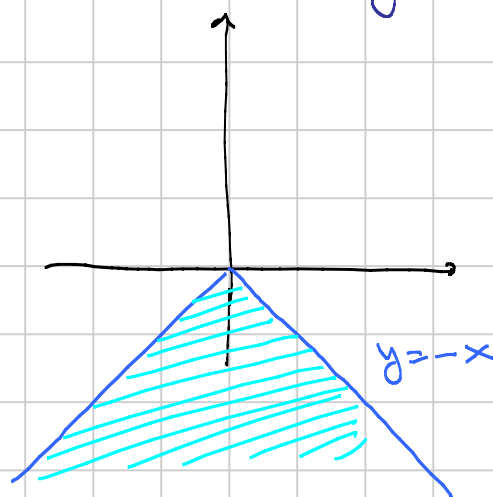


$$\textcircled{2} \{(x, y) \in \mathbb{R}^2 : y - x^2 \geq 0\}$$



$$\textcircled{3} \{(x, y) \in \mathbb{R}^2 : y + |x| \leq 0\}$$

$$y \leq -|x|$$

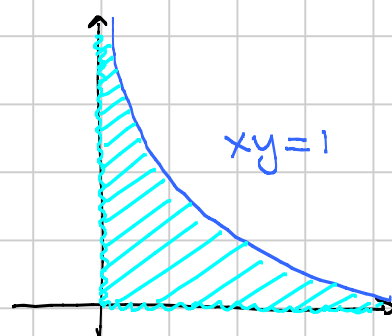


$$\textcircled{4} \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0, xy \leq 1\}$$

$\underbrace{x \geq 0, y \geq 0}_{1^\circ \text{ quadr.}}$

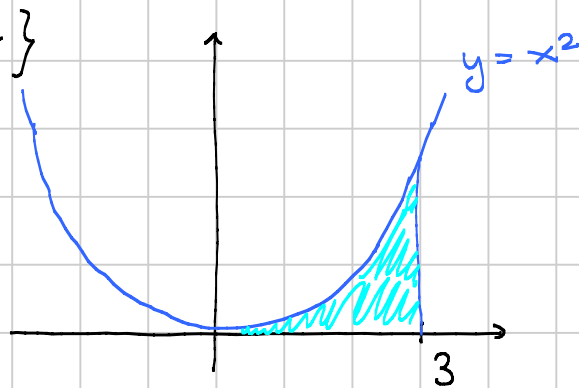
$$\uparrow y \leq \frac{1}{x}$$

(Posso dividere
perché $x > 0$)



$$\textcircled{5} \{ (x,y) \in \mathbb{R}^2 : 0 \leq x \leq 3, 0 \leq y \leq x^2 \}$$

x compreso tra 0 e 3
 y sopra 0 e sotto x^2



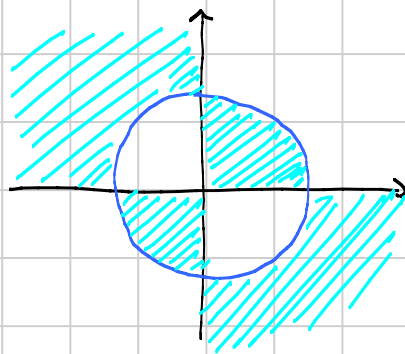
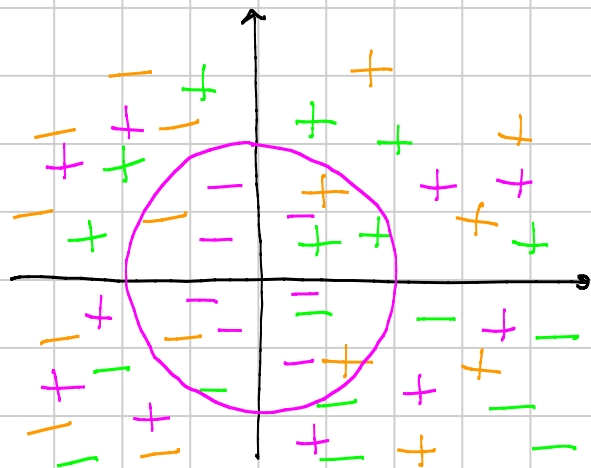
$$\textcircled{6} \{ (x,y) \in \mathbb{R}^2 : xy(x^2+y^2-1) \leq 0 \}$$

x ●
 y ●
 x^2+y^2-1 ●

$$x^2+y^2-1 \geq 0 \Leftrightarrow x^2+y^2 \geq 1$$

Ci interessano le zone con

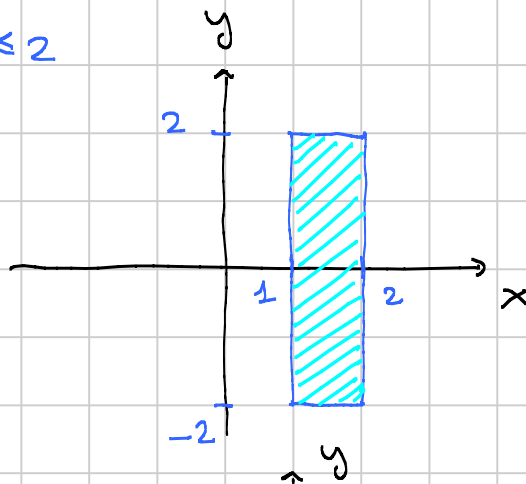
- 3 segni -
- ++ -



$$\textcircled{7} \{ (x,y) \in \mathbb{R}^2 : \underbrace{x^2-3x+2 \leq 0}, \underbrace{y^2-4 \leq 0} \}$$

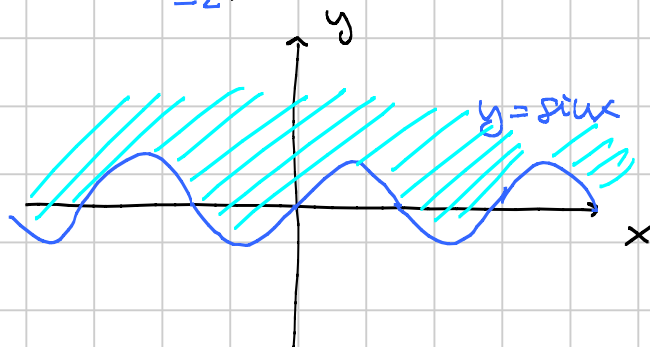
$$\begin{aligned} (x-2)(x-1) &\leq 0 \\ 1 &\leq x \leq 2 \end{aligned}$$

$$\begin{aligned} y^2 &\leq 4 \\ -2 &\leq y \leq 2 \end{aligned}$$



$$\textcircled{8} \{ (x,y) \in \mathbb{R}^2 : \sin x - y \leq 0 \}$$

$$y \geq \sin x$$

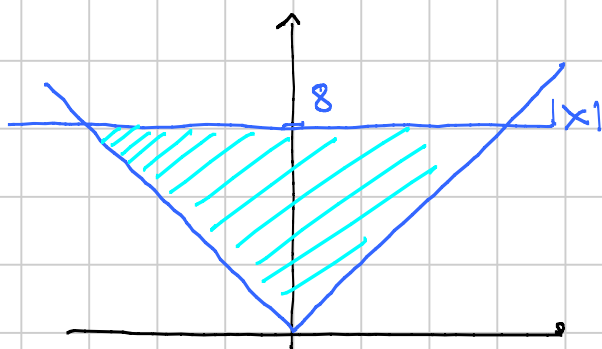


$$\textcircled{9} \{ (x,y) \in \mathbb{R}^2 : |x| \leq y \leq 8 \}$$

Due condizioni vere contempor.

$$y \leq 8$$

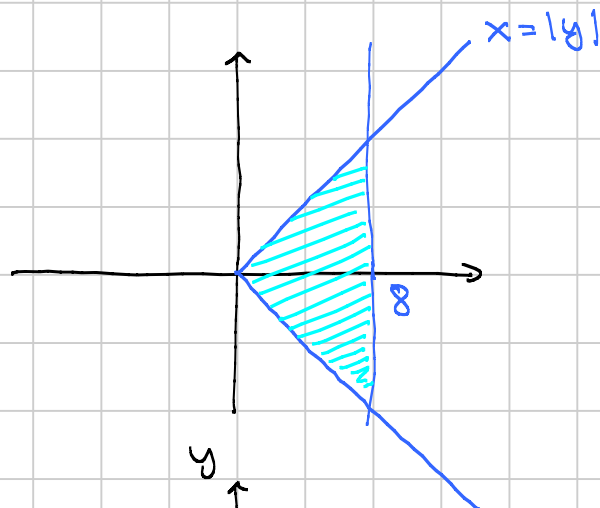
$$y \geq |x|$$



$$\textcircled{10} \{ (x,y) \in \mathbb{R}^2 : |y| \leq x \leq 8 \}$$

$$x \leq 8$$

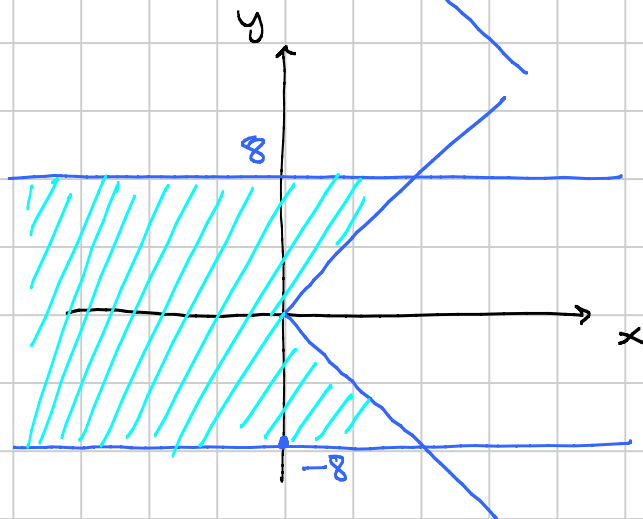
$$x \geq |y|$$



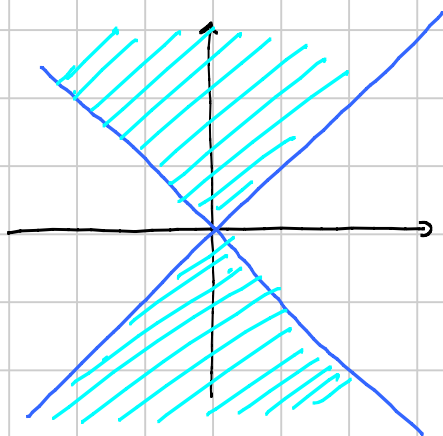
$$\textcircled{11} \{ (x,y) \in \mathbb{R}^2 : x \leq |y| \leq 8 \}$$

$$x \leq |y| \quad (x \text{ a sinistra di } |y|)$$

$$|y| \leq 8 \quad (-8 \leq y \leq 8)$$

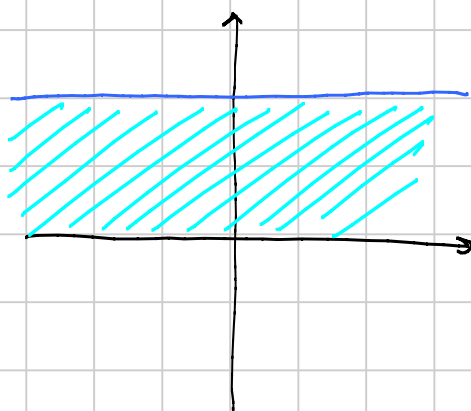


$$\textcircled{12} \{ (x,y) \in \mathbb{R}^2 : x^2 - y^2 \leq 0, y^2 - 4y \leq 0 \}$$



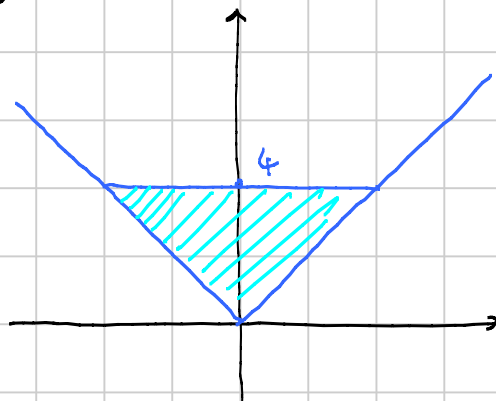
$$x^2 - y^2 \leq 0$$

$$(x-y)(x+y) \leq 0$$



$$y^2 - 4y \leq 0$$

$$0 \leq y \leq 4$$



Avrei ottenuto lo stesso $\{ (x,y) \in \mathbb{R}^2 : |x| \leq y \leq 4 \}$

13) IMPORTANTE $\{(x,y) \in \mathbb{R}^2: |x| + |y| \leq 2\} = A$

Bisognerebbe considerare 4 casi a seconda del segno di x e y

1° quadrante: $x+y \leq 2, y \leq 2-x$

Ora possiamo procedere in due modi

1° modo Trattare allo stesso modo gli altri quadranti

2° quadrante: $|x| + |y| \leq 2 \Leftrightarrow -x + y \leq 2 \quad y \leq x+2$

3° " : " $\Leftrightarrow -x - y \leq 2 \quad y \geq -x-2$

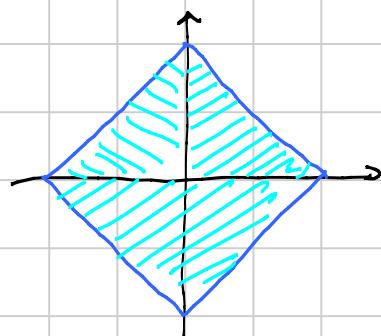
4° " : $|x| + |y| \leq 2 \Leftrightarrow x - y \leq 2 \quad y \geq x-2$

2° modo Osservare che l'insieme è simmetrico risp. agli assi

$(x,y) \in A \Leftrightarrow (-x,y) \in A$ (simmetrico risp. asse y)

$(x,y) \in A \Leftrightarrow (x,-y) \in A$ (simmetria " " x)

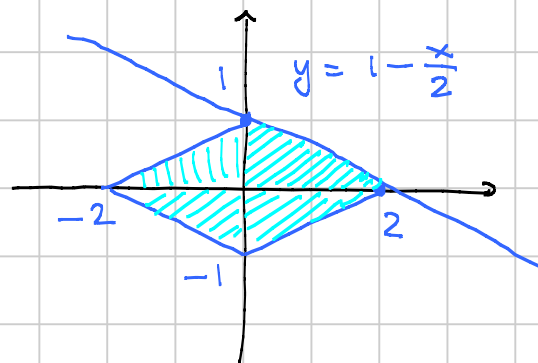
Una volta capito il 1° quadrante, basta "ribaltare"



14) $\{(x,y) \in \mathbb{R}^2: |x| + 2|y| \leq 2\}$

1° quadrante: $x+2y \leq 2$
 $y \leq 1 - \frac{x}{2}$

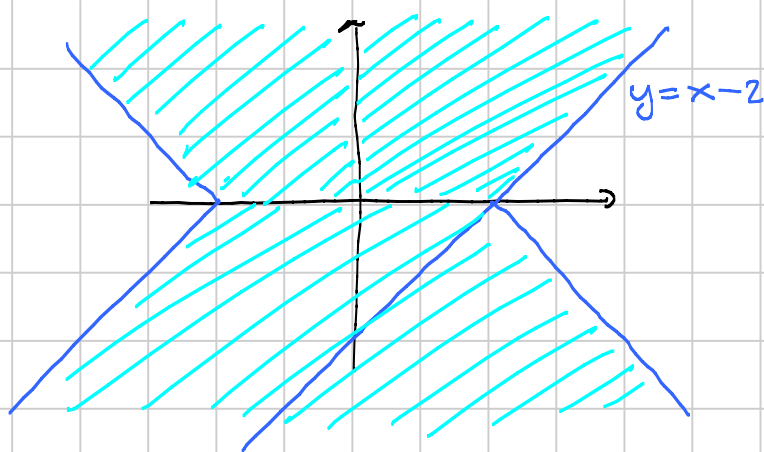
Replico per simmetria



$$(15) \{ (x,y) \in \mathbb{R}^2 : |x| - |y| \leq 2 \}$$

1° quadrante: $x - y \leq 2$
 $y \geq x - 2$

Ripeto per simmetria



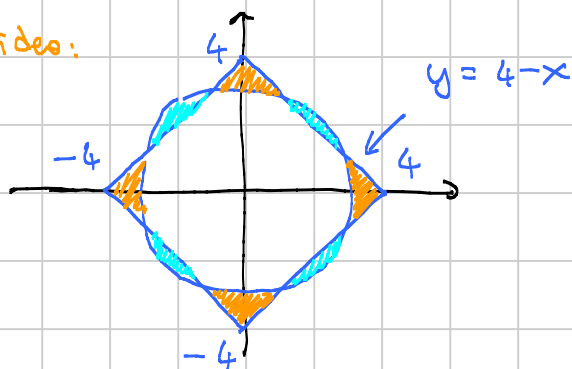
[Per esercizio: dividere i casi nei 4 quadranti!]

$$(16) \{ (x,y) \in \mathbb{R}^2 : |x| + |y| \leq 4, x^2 + y^2 \geq 9 \}$$

dentro al
rombo

fuori dal
cerchio di raggio 3

[Conetto dopo video:
La zona è
quella arancio,
ovviamente!]



Quattro zone laterali

Cerchio e rombo si
intersecano?

$$\begin{cases} x^2 + y^2 = 9 \\ y = 4 - x \end{cases}$$

$$x^2 + 16 - 8x + x^2 = 9$$

$$2x^2 - 8x + 7 = 0$$

$$\Delta = 16 - 14 > 0$$

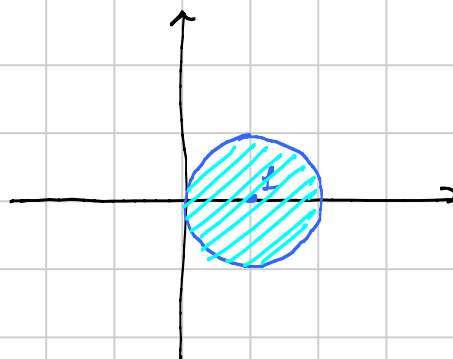
si intersecano!

$$(17) \{ (x,y) \in \mathbb{R}^2 : x^2 - 2x + y^2 \leq 0 \}$$

$$x^2 - 2x + 1 + y^2 \leq 1$$

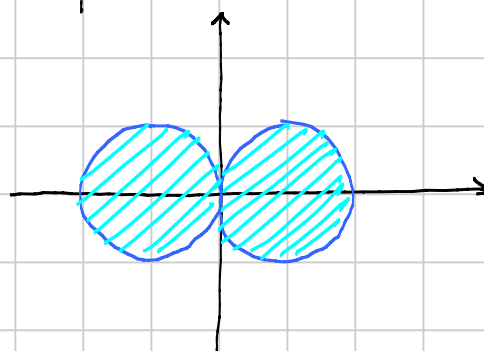
$$(x-1)^2 + y^2 \leq 1$$

distanza da (1,0) ≤ 1



$$(18) \{ (x,y) \in \mathbb{R}^2 : x^2 - 2|x| + y^2 \leq 0 \}$$

Come prima per $x \geq 0$, poi simmetrico
rispetto a $x = 0$.



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