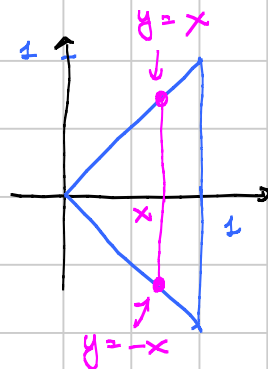


Integrali di funzioni con valori assolutiEsempio 1 $A =$ triangolo con vertici $(0,0)$, $(1,1)$, $(1,-1)$

$$\iint_A |x| dx dy = \iint_A x dx dy = \int_0^1 dx \int_{-x}^x dy \quad x =$$

$x \geq 0$ in A , quindi
posso togliere il valore assoluto

$$= \int_0^1 x dx \int_{-x}^x dy = \int_0^1 2x^2 dx = \frac{2}{3}.$$

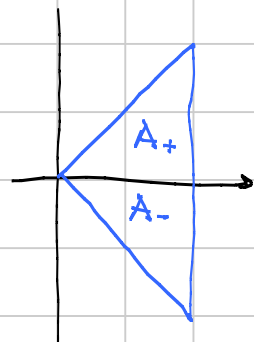
Esempio 2 A come prima

$$\iint_A |y| dx dy = 2 \iint_{A_+} y dx dy$$

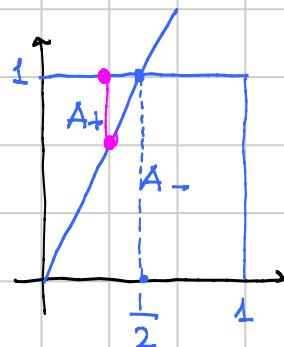
$\uparrow$
per simmetria: l'integrale
su A_+ e su A_- è lo stesso

$$= 2 \int_0^1 dx \int_0^x dy y = \int_0^1 dx [y^2]_{y=0}^{y=x} = \int_0^1 x^2 dx = \frac{1}{3}.$$

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Esempio 3 $A = [0,1] \times [0,1]$

$$\iint_A |y-2x| dx dy$$



Conco di capire la zona A_+ dove $y-2x \geq 0$
e la zona A_- dove $y-2x \leq 0$

$$y \geq 2x$$

$$y \leq 2x$$

$$= \iint_{A_+} (y-2x) dx dy + \iint_{A_-} (2x-y) dx dy$$

$\uparrow$
in A_+ il val. assol.
è come se non ci fosse

$\uparrow$
in A_- si ha che
 $|y-2x| = -(y-2x)$

$$\begin{aligned}\iint_{A_+} (y-2x) dx dy &= \int_0^{\frac{1}{2}} dx \int_0^{2x} dy (y-2x) = \int_0^{\frac{1}{2}} dx \left[\frac{1}{2} y^2 - 2xy \right]_{y=0}^{y=2x} \\ &= \int_0^{\frac{1}{2}} dx (2x^2 - 4x^2) = \int_0^{\frac{1}{2}} (-2x^2) dx\end{aligned}$$

viene negativo e non può essere perché sto integrando una funzione positiva. Quindi qualcosa non va

Ho sbagliato l'insieme!

Correggo:

$$\begin{aligned}\iint_{A_+} (y-2x) dx dy &= \int_0^{\frac{1}{2}} dx \int_{2x}^1 dy (y-2x) = \int_0^{\frac{1}{2}} dx \left[\frac{1}{2} y^2 - 2xy \right]_{y=2x}^{y=1} \\ &= \int_0^{\frac{1}{2}} dx \left(\frac{1}{2} - 2x - 2x^2 + 4x^2 \right) = \int_0^{\frac{1}{2}} \left(2x^2 - 2x + \frac{1}{2} \right) dx = \dots\end{aligned}$$

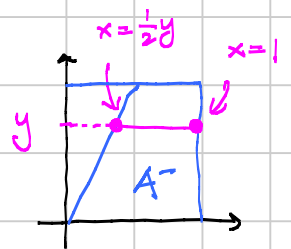
Resta da calcolare l'integrale in A_- .

1° modo: triangolo + rettangolo

$$\iint_{A_-} (2x-y) dx dy = \int_0^{\frac{1}{2}} dx \int_0^{2x} dy (2x-y) + \int_{\frac{1}{2}}^1 dx \int_0^1 dy (2x-y) = \dots$$

2° modo: normale risp. asse y

$$\iint_{A_-} (2x-y) dx dy = \int_0^1 dy \int_{\frac{1}{2}y}^1 dx (2x-y)$$



$$= \int_0^1 dy \left[x^2 - xy \right]_{x=\frac{1}{2}y}^{x=1} = \int_0^1 dy \left(\frac{1-y}{2} - \frac{1}{4}y^2 + \frac{1}{2}y^2 \right)$$

$$= \int_0^1 \left(\frac{1}{4}y^2 - y + 1 \right) dy = \dots$$

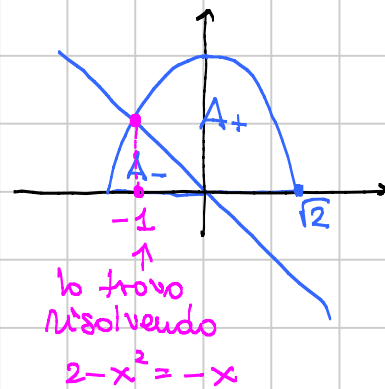
Esercizio: controllare che venga lo stesso nei 2 modi

Esempio 4 $A = \{(x, y) \in \mathbb{R}^2 : 0 \leq y \leq 2 - x^2\}$

$$\iint_A |x+y| dx dy$$

$A_+ =$ zona dove $x+y \geq 0$, cioè $y \geq -x$

$A_- =$ " " $x+y \leq 0$, cioè $y \leq -x$



$$\iint_A |x+y| dx dy = \iint_{A_+} (x+y) dx dy + \iint_{A_-} (-x-y) dx dy$$

$$= \iint_{A_+} (x+y) dx dy - \iint_{A_-} (x+y) dx dy$$

$$\iint_{A_+} (x+y) dx dy = \underbrace{\int_{-1}^0 dx \int_{-x}^{2-x^2} dy (x+y)}_{\text{parte a sx dell'asse y}} + \underbrace{\int_0^{\sqrt{2}} dx \int_0^{2-x^2} dy (x+y)}_{\text{parte a dx dell'asse y}} = \dots$$

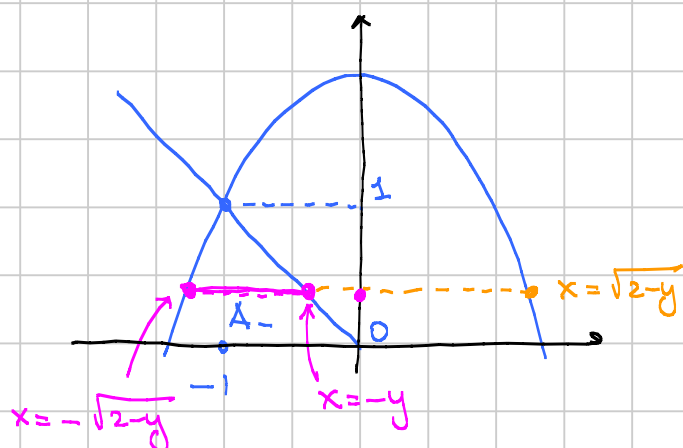
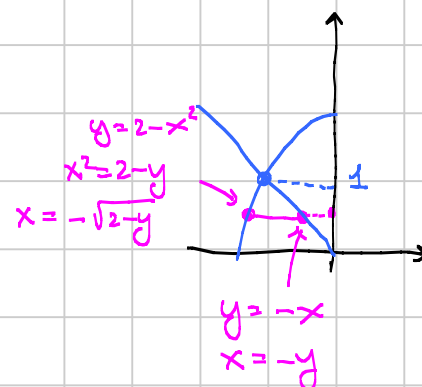
Integrale su A_- , primo modo (normale risp. asse x)

$$\iint_{A_-} (x+y) dx dy = \underbrace{\int_{-\sqrt{2}}^{-1} dx \int_0^{2-x^2} dy (x+y)}_{\text{shaded triangle}} + \underbrace{\int_{-1}^0 dx \int_0^{-x} dy (x+y)}_{\text{shaded triangle}} = \dots$$

Integrale su A_- , secondo modo (normale risp. asse y)

$$\iint_{A_-} (x+y) dx dy = \int_0^1 dy \int_{-\sqrt{2-y}}^{-y} (x+y) dx$$

Controllare che vengano uguali



$$y = 2 - x^2 \quad x^2 = 2 - y \\ x = \pm \sqrt{2 - y}$$

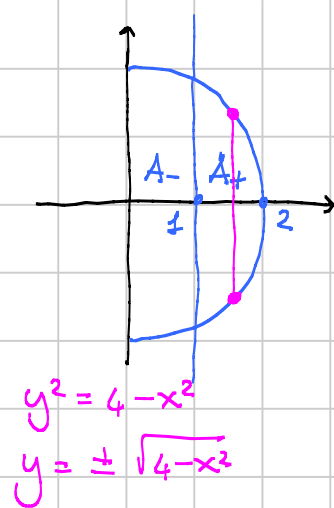
Esempio 5 $A = \{ (x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 4, x \geq 0 \}$

$$\iint_A |x-1| dx dy = \iint_{A_+} (x-1) dx dy - \iint_{A_-} (x-1) dx dy$$

$A_+ = \text{zona con } x \geq 1$

$A_- = \text{zona con } x \leq 1$

$$\begin{aligned} \iint_{A_+} (x-1) dx dy &= \int_1^2 dx \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dy (x-1) \\ &= \int_1^2 dx (x-1) \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dy = 2 \int_1^2 (x-1) \sqrt{4-x^2} dx \\ &= \text{con un po' di lavoro si fa.} \end{aligned}$$



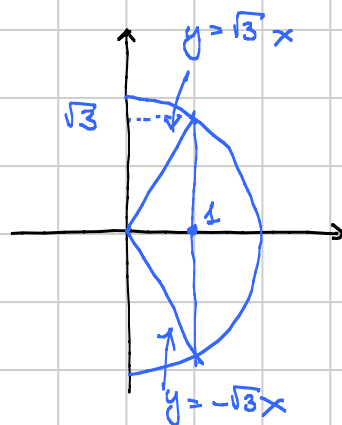
$S = \text{tutto il settore circolare}$

$T = \text{triangolo}$

$\iint_{A_+} \dots = \iint_S - \iint_T$ e i due pezzi si calcolano facilmente

$$\iint_T (x-1) dx dy = \int_0^1 dx \int_{-\sqrt{3}x}^{\sqrt{3}x} dy (x-1) = \text{questo viene}$$

$$\iint_S (x-1) dx dy = \underbrace{\int_0^2 dp \int_{-\pi/2}^{\pi/2} d\theta}_{S \text{ in coord. polari}} \underbrace{(p \cos \theta - 1)}_{x-1 \text{ in coord. polari}} \underbrace{p}_{J} = \text{e questo viene}$$



Anche l'integrale su A_- si può spezzare.

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