

ANALISI II CIV (NUCL)

Titolo nota

ORA 38

29/11/2006

$$\int_0^{+\infty} \underbrace{\frac{\sin x}{x}}_{f(x)} dx$$

unico problema
è a $+\infty$

$f(x)$ è a segno variabile $\Rightarrow$ provo a
mettere i valori assoluti

$$\frac{|\sin x|}{x} \leq \frac{1}{x}$$

$\int \frac{1}{x} dx$ con pb. a $+\infty$ diverge $\Rightarrow$ boh!!!

1^a osservazione: $\int_0^{+\infty} \frac{\sin x}{x} dx$ converge $\Leftrightarrow \int_1^{+\infty} \frac{\sin x}{x} dx$ conv.

$$\int_1^{+\infty} \frac{\sin x}{x} dx = \lim_{A \rightarrow +\infty} \int_1^A \frac{\sin x}{x} dx = \lim_{A \rightarrow +\infty} \int_1^A \underbrace{\frac{1}{x}}_G \underbrace{\sin x}_f dx$$

$$= \lim_{A \rightarrow +\infty} \left[\underbrace{\left[\frac{1}{x} (-\cos x) \right]}_{GF} \Big|_{x=1}^{x=A} - \int_1^A \underbrace{\left(-\frac{1}{x^2} \right)}_g \underbrace{(-\cos x)}_F dx \right]$$

$$= \lim_{A \rightarrow +\infty} \left\{ \underbrace{-\frac{\cos A}{A}}_0 + \frac{\cos 1}{1} - \int_1^A \frac{\cos x}{x^2} dx \right\} =$$

$$= \frac{\cos 1}{1} - \underbrace{\int_1^{+\infty} \frac{\cos x}{x^2} dx}_{\text{CONVERGE}} \in \mathbb{R}$$

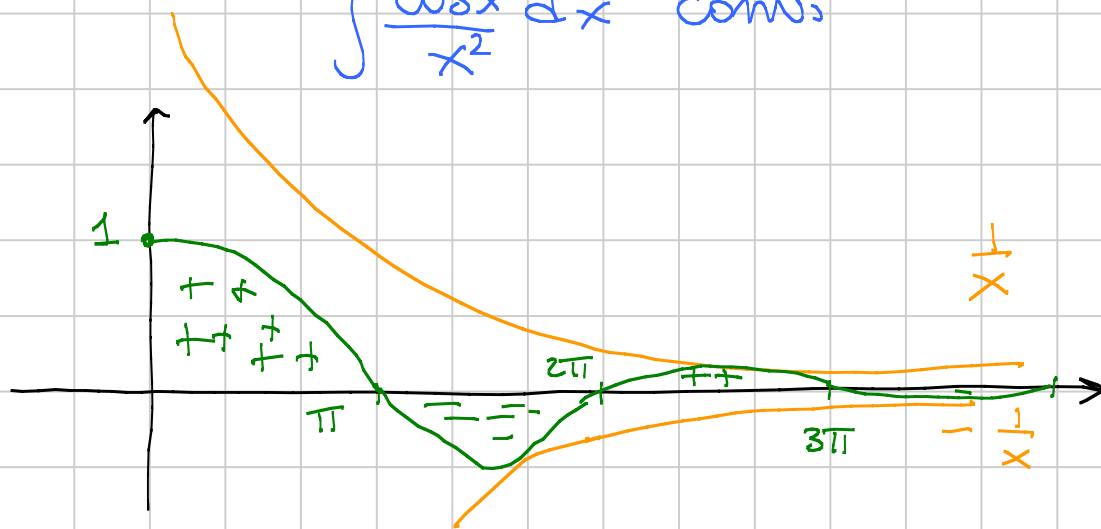
CONFRONTO

$\int \frac{1}{x^2} dx$ con pb. a $+\infty$ conv. $\Rightarrow \int \frac{|\cos x|}{x^2} dx$ con pb. a $+\infty$

$\Downarrow \leftarrow$ Assol. conv.

$$\int \frac{\cos x}{x^2} dx \text{ conv.}$$

$$\int_0^{+\infty} \frac{\sin x}{x} dx \text{ converge}$$



Si potrebbe dimostrare che

$$\int_0^{+\infty} \frac{|\sin x|}{x} dx = +\infty$$

$$\text{Area } \oplus + \text{Area } \ominus = +\infty$$

Facciamo $\lim_{A \rightarrow +\infty} \int_0^A \dots$

Le aree $\oplus$ e quelle $\ominus$ tendono a compensarsi

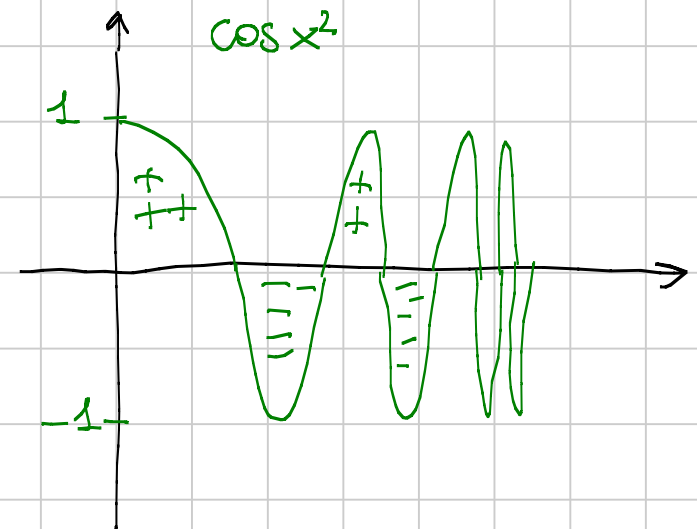
— 0 — 0 —

$$\int_0^{+\infty} \cos(x^2) dx$$

Si potrebbe dimostrare che

$$\int_0^{+\infty} |\cos(x^2)| dx = +\infty$$

$$\int_1^{+\infty} \cos(x^2) dx = \lim_{A \rightarrow +\infty} \int_1^A \underbrace{\frac{1}{2x}}_G \underbrace{2x \cos(x^2)}_F dx =$$



$$= \lim_{A \rightarrow +\infty} \left\{ \left[\frac{1}{2x} \sin(x^2) \right]_{x=1}^{x=A} - \int_1^A \left(-\frac{1}{2x^2} \right) \sin x^2 dx \right\}$$

G F g F

$$= \lim_{A \rightarrow +\infty} \left\{ \frac{\sin A^2}{2A} - \frac{\sin 1}{2} + \frac{1}{2} \int_1^A \frac{\sin x^2}{x^2} dx \right\}$$

↓
0

$$= -\frac{\sin 1}{2} + \frac{1}{2} \int_1^{+\infty} \frac{\sin x^2}{x^2} dx \in \mathbb{R} \Rightarrow \int_0^{+\infty} \cos x^2 dx \text{ conv.}$$

$$\int \frac{dx}{x^2} \text{ con pb. a } +\infty \text{ conv.} \Rightarrow \int \frac{|\sin x^2|}{x^2} dx \text{ con pb. a } +\infty \text{ conv.}$$

↓

$$\int \frac{\sin x^2}{x^2} dx \text{ con pb. a } +\infty \text{ conv.}$$

Allo stesso modo $\int_0^{+\infty} \sin(x^2) dx$ conv.

$$\int_{-\infty}^{+\infty} \sin x^2 dx = 2 \int_0^{+\infty} \sin x^2 dx$$

↑
FUNZIONE
PARI

— 0 — 0 —

$$\int_3^5 \frac{dx}{x-3} =$$

Con un cambio di variabili si sposta
il problema in 0

$$x-3 = y$$

$$dx = dy$$

$$\begin{array}{l} \text{Quando } x=3, y=0 \\ \text{" } \quad \quad x=5, y=2 \end{array}$$

$$= \int_0^2 \frac{dy}{y} \quad \text{diverge}$$

In generale

$$\int_a^{a+1} \frac{dx}{(x-a)^\alpha}$$

si riconduce a $\int_0^1 \frac{dy}{y^\alpha}$

dunque converge $\Leftrightarrow \alpha < 1$

$$\int_1^3 \frac{dx}{\sqrt{x^2-1}}$$

↑
problema in
 $x=1$

$$\frac{1}{\sqrt{x^2-1}} = \frac{1}{\sqrt{(x-1)(x+1)}}$$

↑
colpevole

$$\sim \frac{1}{\sqrt{x-1}}$$

↓
l'integrale
CONVERGE ($\alpha = \frac{1}{2}$)

Più rigorosamente, facciamo confronto asintotico con

$$g(x) = \frac{1}{\sqrt{x-1}}$$

$$\lim_{x \rightarrow 1^+} \frac{f(x)}{g(x)} = \lim_{x \rightarrow 1^+} \frac{1}{\sqrt{(x-1)(x+1)}} \cdot \cancel{\sqrt{x-1}} = \lim_{x \rightarrow 1^+} \frac{1}{\sqrt{x+1}} = \frac{1}{\sqrt{2}}$$

— 0 — 0 —

$\neq 0 \neq +\infty \Rightarrow$
stesso comp

$$\int_1^5 \frac{dx}{\log x}$$

↑
prob. in $x=1$

Pongo $y = x-1$

Quando $x=1$, ho che $y=0$

" $x=5$, " $y=4$

$$dy = dx$$

$$= \int_0^4 \frac{dy}{\log(1+y)} = +\infty$$

Brutale; $\log(1+y) \sim y$
 ↑
 Taylor

$$\frac{1}{\log(1+y)} \sim \frac{1}{y} \Rightarrow \text{l'integrale DIVERGE}$$

$$\int_{-\infty}^{+\infty} \frac{dx}{\sqrt{|x^4-1|}}$$

Si spezza in 6 parti,
 uguali a 2 a 2

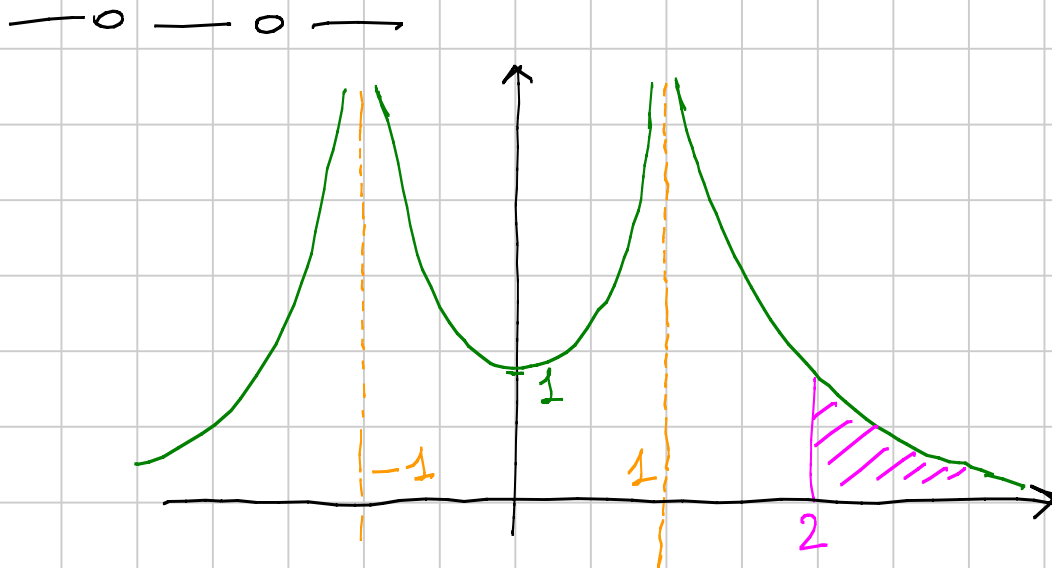
$$\int_2^{+\infty} \frac{dx}{\sqrt{|x^4-1|}}$$

converge

per x grandi

Brutale: $\frac{1}{\sqrt{|x^4-1|}} \sim \frac{1}{\sqrt{x^4}} = \frac{1}{x^2}$

$\int \frac{1}{x^2} dx$ con pb.
 a $+\infty$ converge



$$\int_1^2 \frac{dx}{\sqrt{|x^4-1|}} = \int_1^2 \frac{dx}{\sqrt{x^4-1}}$$

Brutale: $\frac{1}{\sqrt{x^4-1}} = \frac{1}{\sqrt{(x^2+1)(x^2-1)}} = \frac{1}{\sqrt{(x^2+1)(x+1)(x-1)}}$

↑
COLPEVOLE

Quindi l'integrale

$$\int_1^2 \frac{dx}{\sqrt{|x^4-1|}} \text{ si comporta come } \int_1^2 \frac{dx}{\sqrt{x-1}}, \text{ quindi conv.}$$

Anche $\int_0^1 \dots$ converge per lo stesso motivo.

$$\Rightarrow \int_{-\infty}^{\infty} \dots \text{ converge}$$