

ANALISI II CIV

ORA 35

BARICENTRI - GULDINO - ANALITICA NELLO SPAZIO

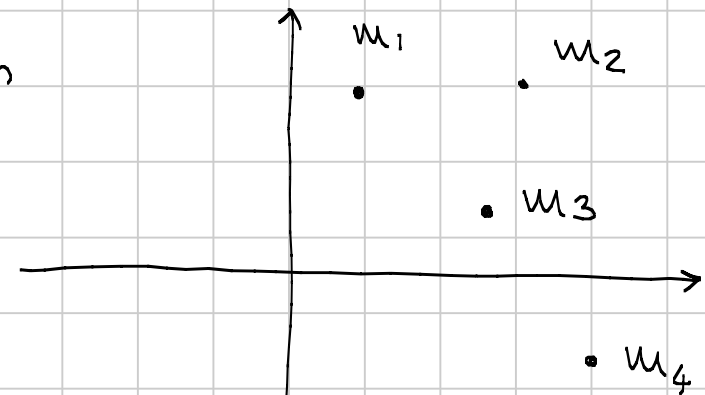
Fisica : punti $P_i (x_i, y_i)$ $i=1, \dots, n$

massa m_i

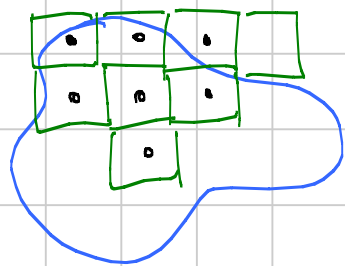
$$M = m_1 + \dots + m_n = \text{massa totale}$$

$$\text{Baricentro : } \left(\frac{1}{M} (m_1 x_1 + \dots + m_n x_n), \frac{1}{M} (m_1 y_1 + \dots + m_n y_n) \right)$$

COORD x del
baricentro = media
pesata delle x
dei singoli punti



↑
stessa cosa
con la y



Si divide in quadratini, faccio finta che ogni quadratino sia un punto (centro) e mi comporto come se fossero un numero finito di punti. Poi infittisco i quadratini

Alla fine si ottiene che il baricentro di una figura piana $F \subseteq \mathbb{R}^2$ è il punto $G = (x_G, y_G)$

$$x_G = \frac{1}{\text{Area}(F)} \iint_F x \, dx \, dy$$

$$y_G = \frac{1}{\text{Area}(F)} \iint_F y \, dx \, dy$$

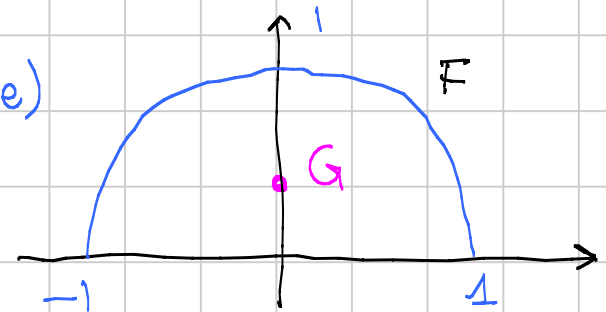
Fisicamente stiamo supponendo F costruita con un materiale omogeneo

Il baricentro di un solido S è il punto $G = (x_G, y_G, z_G)$ con

$$x_G = \frac{1}{\text{Vol}(S)} \iiint_S x \, dx \, dy \, dz \quad \text{e analogo per } y_G \text{ e } z_G$$

Esempio 1 Baricentro di un semicerchio (raggio 1)

$x_G = 0$ per simmetria (verificare l'integrale)



$$\text{Area} = \frac{\pi}{2} \quad \left(\frac{\pi R^2}{2} \right)$$

$$\iint_F y \, dx \, dy = \int_0^1 dp \int_0^\pi \underbrace{p \sin \theta}_y \underbrace{p}_J =$$

↑
COORD
POLAR)

$$= \int_0^1 p^2 dp \underbrace{\int_0^\pi \sin \theta \, d\theta}_2 = 2 \int_0^1 p^2 dp = \frac{2}{3}$$

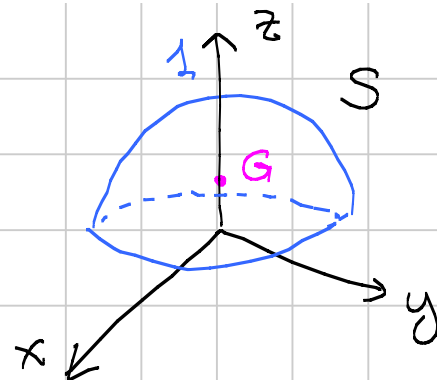
Di conseguenza $y_G = \frac{1}{\text{Area}} \iint y = \frac{2}{\pi} \cdot \frac{2}{3} = \frac{4}{3\pi}$

Esempio 2

Semisfera

$$G = (0, 0, z_G)$$

per simmetria



$$\text{Vol}(S) = \frac{1}{2} \frac{4}{3} \pi = \frac{2}{3} \pi$$

COORD. SFERICHE

$$\iiint_S z \, dx \, dy \, dz \stackrel{\downarrow}{=} \int_0^1 \rho \, d\rho \int_0^{2\pi} d\theta \int_0^{\pi/2} d\varphi \underbrace{\rho \sin \varphi}_z \rho^2 \cos \varphi$$

$$= 2\pi \int_0^1 \rho^3 \, d\rho \int_0^{\pi/2} \underbrace{\frac{2 \sin \varphi \cos \varphi}{2}}_{\frac{\sin(2\varphi)}{2}} d\varphi$$

$$= 2\pi \int_0^1 \rho^3 \, d\rho \left[-\frac{\cos(2\varphi)}{4} \right]_{\varphi=0}^{\varphi=\frac{\pi}{2}} \quad \frac{1}{2}$$

$$= \pi \int_0^1 \rho^3 \, d\rho = \frac{\pi}{4} \Rightarrow z_G = \frac{1}{\text{Vol}} \cdot \iiint_S z$$

$$= \frac{3}{2\pi} \cdot \frac{\pi}{4} = \boxed{\frac{3}{8}}$$

$$\frac{3}{8} < \frac{4}{3\pi} ;$$

$$3\pi < 32$$

OK

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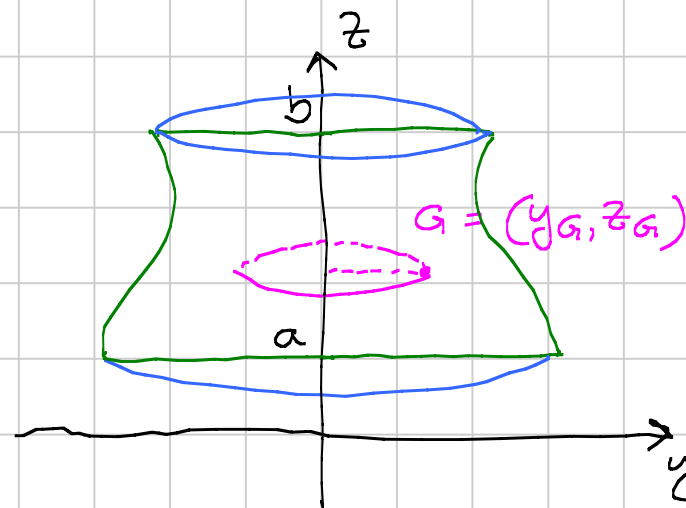
TEO. GULDINO

Sia S il solido di rotazione ottenuto ruotando una figura F (nel piano yz) attorno all'asse z . Allora

$$\text{Vol}(S) = \text{Area}(F) \cdot 2\pi y_G$$

↑
Area
Figura

↑
lunghezza
della circ.
descritta da
 G durante
la rotazione



$$y_G = \frac{1}{\text{Area}(F)} \cdot \iint_F y \, dy \, dz = \frac{1}{\text{Area}(F)} \int_a^b dz \int_0^{\varphi(z)} y \, dy =$$

↑
NORMALE
ASSE z

$$= \frac{1}{\text{Area}(F)} \int_a^b dz \left[\frac{y^2}{2} \right]_{y=0}^{y=\varphi(z)} = \frac{1}{\text{Area}(F)} \frac{1}{2} \int_a^b \varphi^2(z) dz$$

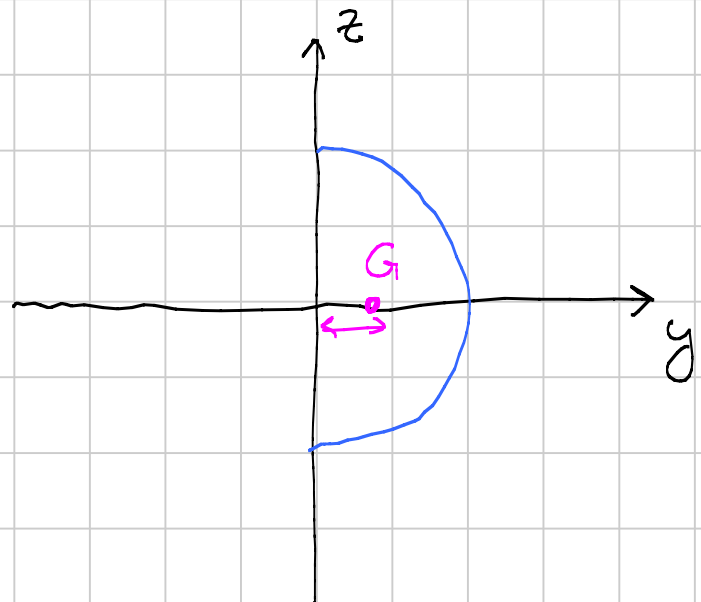
Conseguenza:

$$\begin{aligned} \text{Area}(F) \cdot 2\pi y_G &= \cancel{\text{Area}(F)} \cdot \cancel{2\pi} \frac{1}{\cancel{\text{Area}(F)}} \cdot \cancel{\frac{1}{2}} \int_a^b \varphi^2(z) dz \\ &= \pi \int_a^b \varphi^2(z) dz = \text{Vol}(S), \quad \square \end{aligned}$$

Esempio 1

Volume (Sfera) =

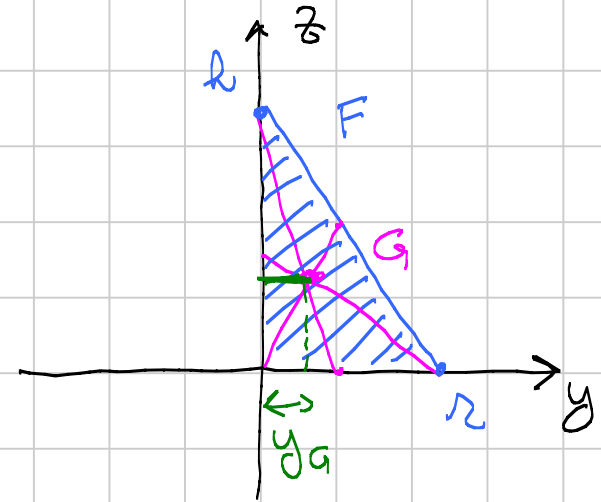
$$\begin{aligned} &= \text{Area}(F) \cdot 2\pi y_G \\ &= \frac{\pi}{2} \cdot \cancel{2\pi} \cdot \frac{4}{3\pi} = \frac{4}{3} \pi \end{aligned}$$



Esempio 2 Cono

$$\text{Volume (Cone)} = \text{Area (P)} \cdot 2\pi y_G$$

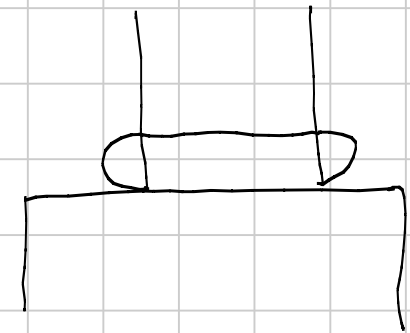
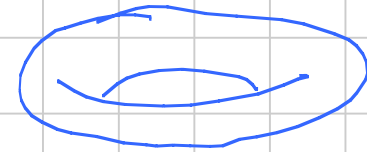
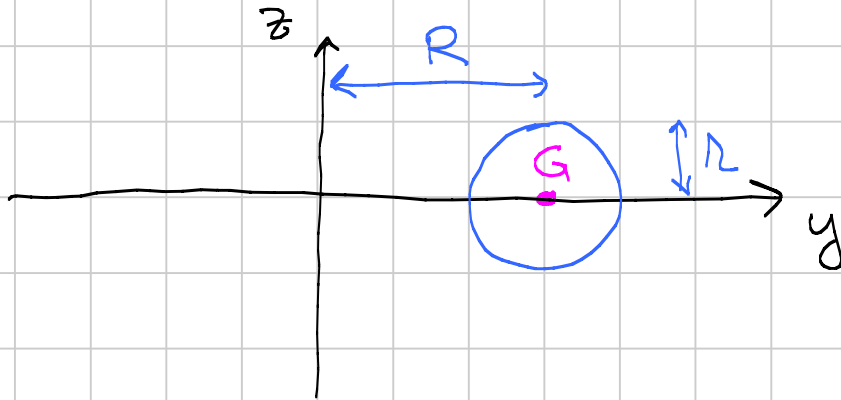
$$\frac{\pi R^2 R}{3} = \frac{1}{2} R R \cdot 2\pi \cdot \frac{R}{3}$$



In un triangolo le coord. del baricentro sono la media delle coord. dei vertici.

Conseguenza: $y_G = \frac{R}{3}$

Esempio 3 TORO



$$\text{Volume (Toro)} = \text{Area (F)} \cdot 2\pi y_G$$

$$\begin{array}{c} \parallel \\ \pi r^2 \end{array}$$

$$\begin{array}{c} \parallel \\ 2\pi R \end{array}$$

$$= \boxed{2\pi^2 r^2 R}$$