

ANALISI 2 CIV

ORA 33

COORDINATE CILINDRICHE

Cartesiane (x, y, z)

Cilindriche (ρ, θ, z)

z = quota = stessa delle coord. cartesiane

(ρ, θ) = corrispondenti polari di (x, y)

Descrizione di un insieme in coord. cilindriche

A = solito cilindro con raggio base 1, altezza: $[0, 2]$ e asse lungo asse z

$$A : z \in [0, 2], \rho \in [0, 1], \theta \in [0, 2\pi]$$

$$\iiint_A z \, dx \, dy \, dz = \int_0^1 d\rho \int_0^{2\pi} d\theta \int_0^2 dz \cdot z \boxed{\rho} =$$

↑
COORD.
CILINDRICHE

PAGAMENTO

$$\begin{aligned}
 &= \int_0^1 \rho \, d\rho \int_0^{2\pi} d\theta \int_0^2 z \, dz = \int_0^1 \rho \, d\rho \int_0^{2\pi} d\theta \left[\frac{z^2}{2} \right]_{z=0}^{z=2} \\
 &= 2 \int_0^1 \rho \, d\rho \int_0^{2\pi} d\theta = 4\pi \int_0^1 \rho \, d\rho = 4\pi \left[\frac{\rho^2}{2} \right]_{\rho=0}^{\rho=1} = 4\pi \cdot \frac{1}{2}
 \end{aligned}$$

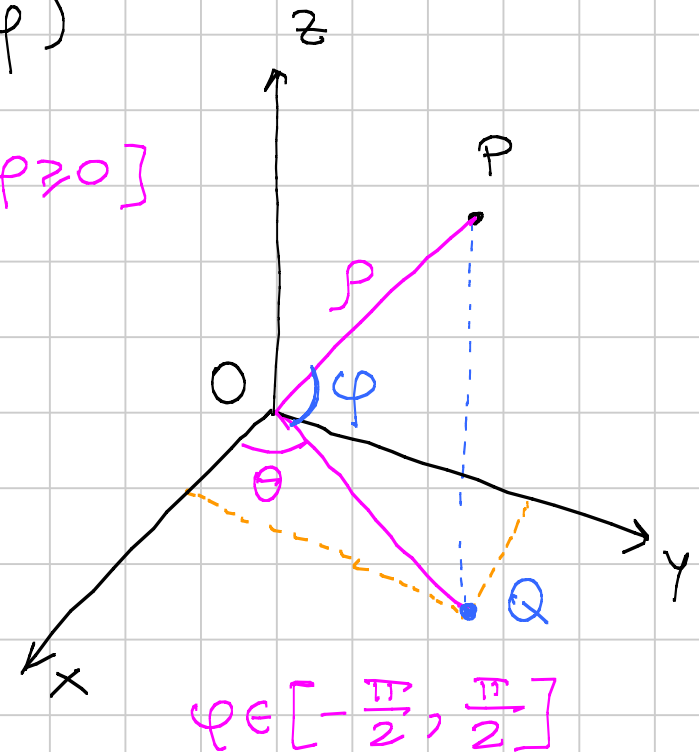
COORDINATE SFERICHE (ρ, θ, φ)

$\rho = OP =$ distanza di P dall'origine $[\rho \geq 0]$

$Q =$ proiezione di P sul piano base xy

$\theta =$ il θ di Q come punto del piano xy $\theta \in [0, 2\pi]$

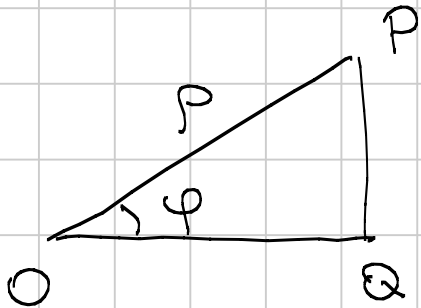
$\varphi =$ angolo tra OP e $OQ =$ angolo tra OP e il piano base xy



In linguaggio geografico : φ = latitudine
 Θ = longitudine misurata
rispetto al meridiano che
passa per il semiasse positivo
delle x .

Formule di passaggio $\rho = \sqrt{x^2 + y^2 + z^2}$

Note ρ, θ, φ , determinare (x, y, z)

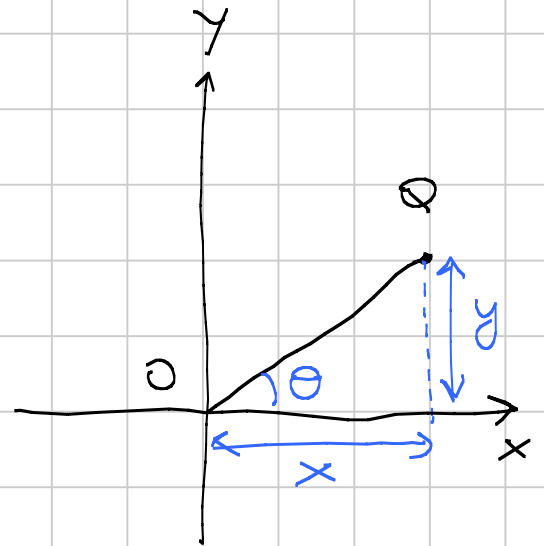


$$z = PQ = \rho \sin \varphi$$

$$OQ = \rho \cos \varphi$$

$$x = OQ \cdot \cos \theta = \rho \cos \varphi \cos \theta$$

$$y = OQ \cdot \sin \theta = \rho \cos \varphi \sin \theta$$



Formule di passaggio:

$$x = \rho \cos \varphi \cos \theta$$

$$y = \rho \cos \varphi \sin \theta$$

$$z = \rho \sin \varphi$$

Descrizione di insiemi in coord. sferiche:

$$A = \{ (x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 4 \}$$

Sfera con centro
in $(0, 0, 0)$ e raggio 2

$$\rho \in [0, 2], \quad \theta \in [0, 2\pi], \quad \varphi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$B = \{ (x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 3, z \geq 0 \}$$

Metà alta della
sfera con centro
in $(0, 0, 0)$ e raggio $\sqrt{3}$

$$\rho \in [0, \sqrt{3}], \quad \theta \in [0, 2\pi], \quad \varphi \in \left[0, \frac{\pi}{2}\right]$$

$$A = \{ (x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 4, x \geq 0, y \geq 0, z \geq 0 \}$$

$\frac{1}{8}$ di sfera
compreso nel

$$\rho \in [0, 2], \quad \theta \in \left[0, \frac{\pi}{2}\right], \quad \varphi \in \left[0, \frac{\pi}{2}\right]$$

I OTTANTE

$$\iiint_B z \, dx \, dy \, dz = \underbrace{\int_0^{\sqrt{3}} \rho \, d\rho \int_0^{2\pi} d\theta \int_0^{\pi/2} d\varphi}_{\text{DESCRIZIONE DI B IN COORD. SFERICHE}} \underbrace{\rho \sin \varphi}_{\text{Z IN COORD. SFERICHE}} \underbrace{\rho^2 \cos \varphi}_{\text{PAGAMENTO}}$$

$$= \int_0^{\sqrt{3}} \rho^3 \, d\rho \int_0^{2\pi} d\theta \int_0^{\pi/2} \frac{2 \sin \varphi \cos \varphi}{2} \, d\varphi$$

$$= \int_0^{\sqrt{3}} \rho^3 \, d\rho \int_0^{2\pi} d\theta \int_0^{\pi/2} \frac{\sin(2\varphi)}{2} \, d\varphi$$

$$\left[-\frac{\cos(2\varphi)}{4} \right]_0^{\pi/2} = -\frac{\cos \pi}{4} + \frac{\cos 0}{4} = \frac{1}{2}$$

$$= \frac{1}{2} \int_0^{\sqrt{3}} \rho^3 \, d\rho \underbrace{\int_0^{2\pi} d\theta}_{2\pi} = \frac{1}{2} \cdot 2\pi \int_0^{\sqrt{3}} \rho^3 \, d\rho = \pi \left[\frac{\rho^4}{4} \right]_0^{\sqrt{3}} = \boxed{\frac{9\pi}{4}}$$

