

ANALISI II CIN (NUCL)

ORA 25

$$\int \frac{dx}{3+x^2} = \int \frac{dx}{1+(2+x^2)} = ?$$

$$\rightarrow = \int \frac{1}{3 \left(1 + \frac{x^2}{3}\right)} dx = \frac{1}{3} \int \frac{1}{1 + \frac{x^2}{3}} dx$$

$$= \frac{\sqrt{3}}{3} \arctan \frac{x}{\sqrt{3}}$$

3 ← $\frac{1}{3}$ che è fuori

In generale

$$\int \frac{dx}{1+ax^2} = \frac{1}{\sqrt{a}} \arctan(\sqrt{a}x) \quad \text{per ogni } a > 0$$

$$\int \frac{dx}{1-x^2} = \int \frac{dx}{(1+x)(1-x)} = (\star)$$

$$\frac{1}{(1+x)(1-x)} = \frac{A}{1+x} + \frac{B}{1-x} = \frac{A-Ax+B+Bx}{(1+x)(1-x)}$$

$$\begin{cases} A+B=1 \\ -A+B=0 \end{cases} \quad \begin{array}{l} \leftrightarrow \text{uguaglianza dei termini noti} \\ \leftrightarrow \text{dei coeff. di } x \end{array}$$

$$2B=1 \quad B=\frac{1}{2} \quad A=\frac{1}{2}$$

$$(\star) = \int \left(\frac{1}{2} \frac{1}{1+x} + \frac{1}{2} \frac{1}{1-x} \right) dx =$$

$$= \frac{1}{2} \int \frac{dx}{1+x} + \frac{1}{2} \int \frac{dx}{1-x} = \frac{1}{2} \log |1+x| - \frac{1}{2} \log |1-x|$$

$$= \log \sqrt{\left| \frac{1+x}{1-x} \right|}$$

$$\int \frac{1}{(x+3)(x^2+1)} dx$$

$$\frac{1}{(x+3)(1+x^2)} = \frac{A}{x+3} + \frac{Bx+C}{1+x^2}$$

costanti sopra
fattori di 1° grado

roba di 1° grado
sopra roba di
2° grado

si risolve il sistema per
trovare A, B, C e si conclude

$$\int \frac{3x+4}{1+x^2} dx = \frac{3}{2} \int \frac{2x}{1+x^2} + 4 \int \frac{dx}{1+x^2}$$

$$= \frac{3}{2} \log(1+x^2) + 4 \arctan x$$

SOSTITUZIONI "EVIDENTI"

$$\int x \cos(x^2) dx = \frac{1}{2} \int (2x) \cos(x^2) = \frac{1}{2} \sin x^2$$

$$\int x^2 e^{-x^3} dx = \frac{1}{3} \int 3x^2 e^{-x^3} dx = \frac{1}{3} (-e^{-x^3})$$

$$= -\frac{1}{3} e^{-x^3}$$

$$\int \frac{dx}{x \log^5 x} \quad \text{"dy"}$$

Pongo $y = \log x$; $\frac{dy}{dx} = \frac{1}{x}$; $dy = \frac{dx}{x}$

$$= \int \frac{dy}{y^5} = \int y^{-5} dy = -\frac{1}{4} y^{-4} = -\frac{1}{4} \frac{1}{\log^4 x}$$

— 0 — 0 —

$$\int \frac{dx}{x \log x} \quad \text{"dy"}$$

Pongo $y = \log x$, $\frac{dy}{dx} = \frac{1}{x}$; $dy = \frac{dx}{x}$

$$= \int \frac{dy}{y} = \log |y| = \log |\log x|$$

— 0 — 0 —

$$\int \sqrt{1-x^2} dx \quad x = \sin y; \quad \frac{dx}{dy} = \cos y; \quad dx = \cos y \cdot dy$$

$$= \int \sqrt{1-\sin^2 y} \cdot \cos y dy = \int \cos^2 y dy$$

$$= \frac{y}{2} + \frac{\sin y \cdot \cos y}{2}$$

$$= \frac{y}{2} + \frac{\sin y \cdot \sqrt{1-\sin^2 y}}{2}$$

$$y = \arcsin x$$

$$= \frac{\arcsin x}{2} + \frac{x \sqrt{1-x^2}}{2}$$

Pericoli: abbiamo detto che

$$\cos y = \sqrt{1-\sin^2 y};$$

$$x = \sin y \Rightarrow y = \arcsin x$$

VIVIAMO PERICOLOSAMENTE, ma alla fine controlliamo
facendo la derivata.

$$\int \frac{1}{1+e^{2x}} dx = \text{Pongo } y = e^{2x} ; \frac{dy}{dx} = e^{2x} \cdot 2 ;$$

$$dy = 2e^{2x} dx$$

$$= \int \frac{1}{2e^{2x}} \cdot \frac{2e^{2x}}{1+e^{2x}} dx \quad dy$$

$$= \int \frac{dy}{2y(1+y)} = \frac{1}{2} \int \frac{dy}{y(1+y)} = \frac{1}{2} \left\{ \int \frac{1}{y} - \int \frac{1}{1+y} \right\} =$$

$$\frac{1}{y(1+y)} = \frac{A}{y} + \frac{B}{1+y} = \frac{1}{y} - \frac{1}{1+y} \quad \left| \begin{array}{l} \frac{1}{2} (\log|y| - \log|1+y|) \\ = \frac{1}{2} \log(e^{2x}) - \log(1+e^{2x}) \\ = x - \log(1+e^{2x}) \end{array} \right.$$

↑
fare il
sistema

In alternativa ;

$$\int \frac{dx}{1+e^{2x}} =$$

$y = 1+e^{2x}$ $\frac{dy}{dx} = 2e^{2x}$ $dy = 2e^{2x} dx$

$$= \int \frac{1}{2e^{2x}} \cdot \frac{2e^{2x}}{1+e^{2x}} dx = \int \frac{dy}{(y-1)y}$$

— 0 — 0 —

$$\int \frac{x}{1+x^4} dx = \frac{1}{2} \int \frac{2x}{1+x^4} dx = \frac{1}{2} \int \frac{2x}{1+(x^2)^2} dx$$
$$= \frac{1}{2} \arctan x^2$$

$$\int \frac{x^2}{1+x^4} dx = + \text{ DIFFICILE}$$

$$\int \frac{x^3}{1+x^4} dx = \frac{1}{4} \log(1+x^4)$$

$$\int \tan x = \int \frac{\sin x}{\cos x} dx = -\log |\cos x| = \log \frac{1}{|\cos x|}$$

$$\int \cancel{\tan x} dx = \int \underset{f}{\sin x} \cdot \underset{G}{\frac{1}{\cos x}} dx$$

$$= \underset{F}{-\cancel{\cos x}} \cdot \underset{G}{\frac{1}{\cancel{\cos x}}} - \int \underset{Fg}{(-\cancel{\cos x})} \left(-\frac{1}{\cancel{\cos^2 x}} \right) (-\sin x) dx$$

$$= -1 + \int \frac{\sin x}{\cos x} dx$$

$$= -1 + \int \cancel{\tan x} dx \Rightarrow 0 = -1$$