

ANALISI II

CIN
NUCL

ORA 24

Titolo nota

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Ripasso di integrali in una variabile

$$\int \cos x \, dx = + \sin x$$

$$\int x^8 \, dx = \frac{x^9}{9}$$

$$\int e^x \, dx = e^x$$

$$\int e^{2x} \, dx = \frac{e^{2x}}{2}$$

$$\int \sin(4x) \, dx = - \frac{\cos(4x)}{4}$$

$$\int \frac{1}{x} \, dx = \log x \quad \text{corretto per } x > 0$$

$$\int \frac{1}{x} \, dx = \log |x| \quad \text{corretto per } x \neq 0$$

La derivata di $\log |x|$ è $\frac{1}{x}$ per ogni $x \neq 0$

Integrazione per parti

$$\int f G = F G - \int F g$$

$$\begin{aligned} F' &= f \\ G' &= g \end{aligned}$$

$$\int \underset{G}{x} \underset{f}{\cos x} dx = \underset{G}{x} \underset{F}{\sin x} - \int \underset{F}{\sin x} \cdot \underset{g}{1} dx = x \sin x + \cos x$$

$$\int \underset{G}{x} \underset{f}{e^x} dx = \underset{G}{x} \underset{F}{e^x} - \int \underset{F}{e^x} \cdot \underset{g}{1} dx = x e^x - e^x$$

$$\int \underset{G}{x^2} \underset{f}{\cos x} dx = \underset{G}{x^2} \underset{F}{\sin x} - \int \underset{F}{\sin x} \cdot \underset{g}{2x} dx$$

$$= x^2 \sin x - 2 \int x \sin x dx$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x$$

$$\begin{aligned}
 \int \underset{G}{x} \underset{f}{\sin x} dx &= \underset{G}{x} \underset{F}{(-\cos x)} - \int \underset{F}{(-\cos x)} \cdot \underset{g}{1} dx \\
 &= -x \cos x + \int \cos x dx = -x \cos x + \sin x
 \end{aligned}$$

Quali primitive vengono bene con questo metodo:

$$\int p(x) e^{ax} dx ; \int p(x) \cos(ax) dx ; \int p(x) \sin(ax) dx$$

$\uparrow$
 polinomio
 della x

A forza di passaggi per parti, il grado di $p(x)$ scende,

— 0 — 0 —

$$\begin{aligned}
 \boxed{\int \log x} &= \int \underset{f}{1} \cdot \underset{G}{\log x} dx = \underset{F}{x} \underset{G}{\log x} - \int \underset{F}{x} \cdot \underset{g}{\frac{1}{x}} dx \\
 &= x \log x - \int 1 dx = \boxed{x \log x - x}
 \end{aligned}$$

$$\boxed{\int \arctan x \, dx} = \int \underset{f}{1} \cdot \underset{G}{\arctan x} \, dx = \underset{F}{x} \underset{G}{\arctan x} - \int \underset{F}{x} \cdot \underset{f}{\frac{1}{1+x^2}} \, dx$$

$$= x \arctan x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx$$

$$\boxed{= x \arctan x - \frac{1}{2} \log(1+x^2)}$$

Idea: scegliere come G qualcosa che "derivando migliora"
 " " f " " "integrando non
 peggiore più di tanto"

GRANDE RITORNO

$$\int \cos^2 x \, dx = \int \underset{G}{\cos x} \cdot \underset{F}{\cos x} \, dx = \underset{G}{\cos x} \cdot \underset{F}{\sin x} - \int \underset{Fg}{\sin x} (-\sin x) \, dx$$

$$= \cos x \cdot \sin x + \int \sin^2 x \, dx$$

$$= \cos x \cdot \sin x + \int (1 - \cos^2 x) \, dx$$

$$= \cos x \cdot \sin x + x - \int \cos^2 x \, dx \quad \rightarrow \text{Lo porto a sinistra}$$

$$2 \int \cos^2 x \, dx = \frac{\cos x \cdot \sin x + x}{2}$$

$$\int \underset{f}{e^x} \underset{g}{\cos x} dx = \underset{F}{e^x} \underset{G}{\cos x} - \int \underset{Fg}{e^x} (-\sin x) dx$$

$$= e^x \cos x + \int \underset{f}{e^x} \underset{g}{\sin x} dx$$

$$= \underset{F}{e^x} \cos x + \underset{G}{e^x} \sin x - \int \underset{Fg}{e^x} \cos x dx$$

lo porto a sinistra

$$2 \int e^x \cos x dx = \frac{e^x \cos x + e^x \sin x}{2}$$

Con lo stesso sistema $\int e^{ax} \sin(bx) dx$

$$\int \sin x \cdot \cos x \, dx = \frac{1}{2} \int 2 \sin x \cdot \cos x \, dx = \frac{1}{2} \int \sin(2x) \, dx$$

$$= \frac{1}{2} (-\cos(2x)) \cdot \frac{1}{2} = -\frac{\cos(2x)}{4}$$

$$\int \cos^2 x \, dx = \int \frac{\cos(2x) + 1}{2} \, dx$$

$$\cos(2x) = 2\cos^2 x - 1$$

$$\cos^2 x = \frac{\cos(2x) + 1}{2}$$

$$= \frac{x}{2} + \frac{1}{2} \int \cos(2x) \, dx$$

$$= \frac{x}{2} + \frac{\sin(2x)}{4} = \frac{x}{2} + \frac{2 \sin x \cdot \cos x}{4} = \text{come prima}$$

$$\int \sin^2 x \, dx = \int \frac{1 - \cos(2x)}{2} \, dx$$

$$\cos(2x) = 1 - 2\sin^2 x$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$= \frac{x}{2} - \frac{1}{2} \int \cos(2x) \, dx = \frac{x}{2} - \frac{\sin(2x)}{4}$$

$$\int \sin^4 x \, dx = \int \underset{f}{\sin x} \cdot \underset{g}{\sin^3 x} \, dx$$

$$= \underset{F}{(-\cos x)} \cdot \underset{G}{\sin^3 x} - \int \underset{F}{(-\cos x)} \underset{g}{3 \sin^2 x \cdot \cos x} \, dx$$

$$= -\cos x \cdot \sin^3 x + 3 \int \cos^2 x \cdot \sin^2 x \, dx$$

$$= -\cos x \cdot \sin^3 x + 3 \int (1 - \sin^2 x) \cdot \sin^2 x \, dx$$

$$= -\cos x \cdot \sin^3 x + 3 \underbrace{\int \sin^2 x \, dx}_{\text{lo so fare}} - 3 \underbrace{\int \sin^4 x \, dx}_{\text{Grande ritorno}}$$

Allo stesso modo si fanno tutte le potenze (interi) di $\sin x$ e $\cos x$ (ogni passaggio per pari l'espon. scende di 2)

$$\int \frac{dx}{3x+2} = \frac{1}{3} \int \frac{3}{3x+2} dx = \frac{1}{3} \log |3x+2|$$

$$\int \frac{dx}{ax+b} = \frac{1}{a} \int \frac{a}{ax+b} dx = \frac{1}{a} \log |ax+b|$$

$$\int \frac{dx}{1+x^2} = \arctan x$$

$$\int \frac{1}{1+4x^2} dx = \int \frac{1}{1+(2x)^2} dx = \frac{1}{2} \int \frac{2}{1+(2x)^2} dx = \frac{1}{2} \arctan(2x)$$

$$\int \frac{1}{1+3x^2} dx = \frac{1}{\sqrt{3}} \arctan(\sqrt{3}x)$$

$$\int \frac{dx}{3+x^2} = ?$$