

INSIEMI NORMALI RISPETTO ASSE y

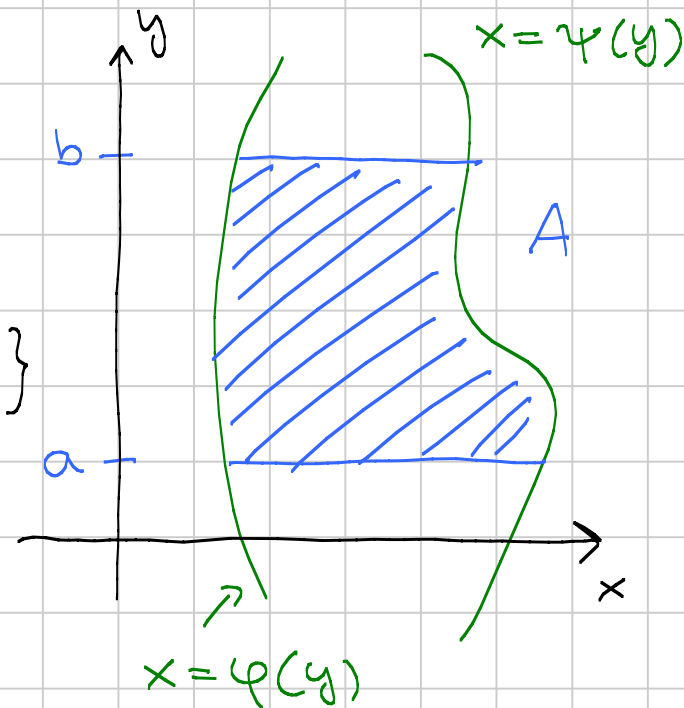
Un insieme si dice normale risp. all'asse y se è della forma

$$A = \{(x, y) \in \mathbb{R}^2 : y \in [a, b], \varphi(y) \leq x \leq \psi(y)\}$$

Per tali insiemi vale la seguente

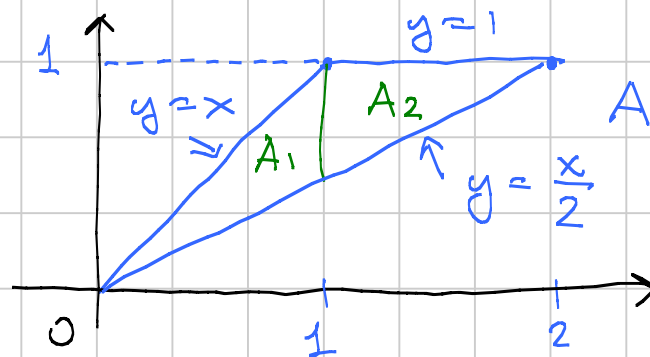
FORMULA DI RIDUZIONE

$$\iint_A f(x, y) dx dy = \int_a^b dy \int_{\varphi(y)}^{\psi(y)} dx f(x, y)$$



Esempio 1

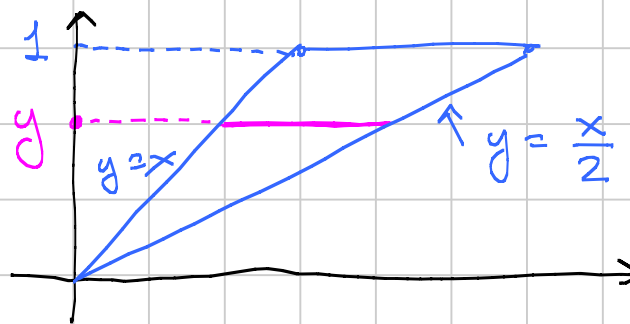
$$\iint_A x \, dx \, dy = \iint_{A_1} x \, dx \, dy + \iint_{A_2} x \, dx \, dy$$



A_1 e A_2 sono normali risp. asse x

$$= \underbrace{\int_0^1 dx \int_{\frac{x}{2}}^x dy}_\text{integr. su } A_1} x + \underbrace{\int_1^2 dx \int_{\frac{x}{2}}^1 dy}_\text{integr. su } A_2} x = \dots \quad \text{così viene}$$

In alternativa si può vedere A come insieme normale risp. asse y



leggo nella
formula
 $x = 2y$

$$A = \{ (x, y) \in \mathbb{R}^2 : y \in [0, 1], \quad y \leq x \leq 2y \}$$

$$\iint_A x \, dx \, dy = \int_0^1 dy \int_y^{2y} dx \, x = \int_0^1 dy \left[\frac{x^2}{2} \right]_{x=y}^{x=2y} =$$

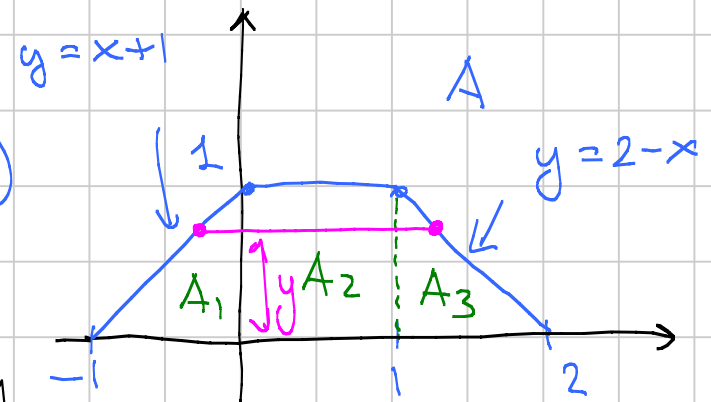
↑
prim. risp. a x

$$= \int_0^1 dy \left\{ \underset{\substack{\uparrow \\ x=2y}}{2y^2} - \underset{\substack{\uparrow \\ x=y}}{\frac{y^2}{2}} \right\} = \frac{3}{2} \int_0^1 y^2 \, dy = \frac{1}{2}$$

Esempio 2

$$\iint_A y \, dx \, dy \quad (> 0 \text{ già sappiamo})$$

$$= \underbrace{\int_{-1}^0 dx \int_0^{x+1} dy \, y}_{\text{su } A_1} + \underbrace{\int_0^1 dx \int_0^1 dy \, y}_{\text{su } A_2} + \underbrace{\int_1^2 dx \int_0^{2-x} dy \, y}_{\text{su } A_3}$$

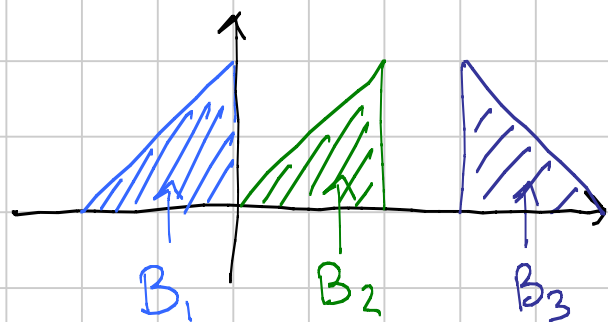


Visto rispetto all'asse y

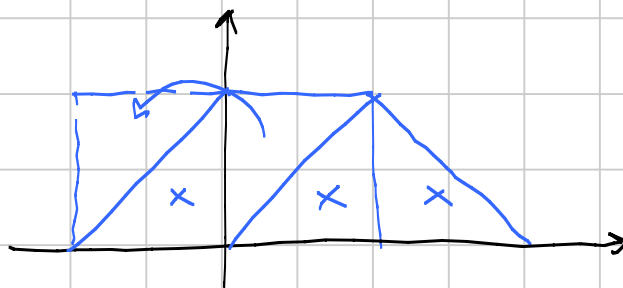
$$A = \{ (x, y) \in \mathbb{R}^2 : y \in [0, 1], \quad y-1 \leq x \leq 2-y \}$$

$$\begin{aligned} \iint_A y \, dx \, dy &= \int_0^1 dy \int_{y-1}^{2-y} dx \cdot \underbrace{y}_{\text{length interval.} = 2-y-(y-1) = 3-2y} = \int_0^1 y \, dy \int_{y-1}^{2-y} dx \\ &= \int_0^1 y(3-2y) \, dy = \text{facile.} \end{aligned}$$

— o — o —



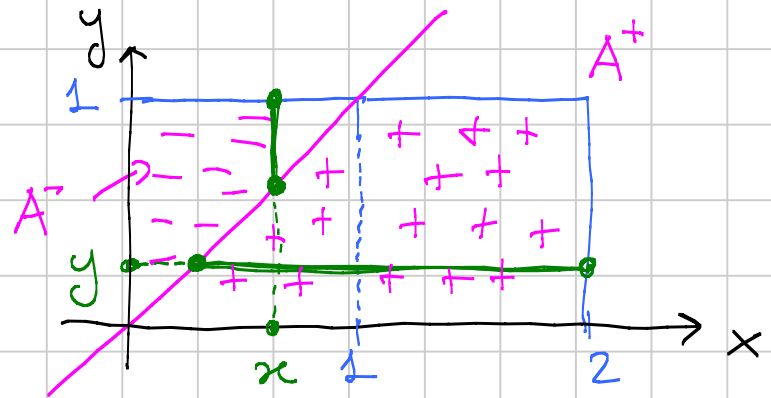
$$\iint_{B_1} y \, dx \, dy = \iint_{B_2} y \, dx \, dy = \iint_{B_3} y \, dx \, dy$$



Esempio 3 $A = [0, 2] \times [0, 1]$ $f(x, y) = |x - y|$

Considero il segno di $x - y$

$$x - y \geq 0 \Leftrightarrow y \leq x$$



$$|x - y| = \begin{cases} x - y & \text{in } A^+ \\ y - x & \text{in } A^- \end{cases}$$

$$\iint_A |x - y| \, dx \, dy = \iint_{A^+} (x - y) \, dx \, dy + \iint_{A^-} (y - x) \, dx \, dy =$$

$$A^- = \left\{ (x, y) \in \mathbb{R}^2 : x \in [0, 1], \quad x \leq y \leq 1 \right\}$$

↑
norm. asse x

$$A^+ = \{ (x, y) \in \mathbb{R}^2 : y \in [0, 1], \quad y \leq x \leq 2 \}$$

↑
norm.
asse y

$$= \int_0^1 dy \int_y^2 dx (x-y) + \int_0^1 dx \int_x^1 dy (y-x)$$

$$= \int_0^1 dy \left[\frac{x^2}{2} - xy \right]_{x=y}^{x=2} + \int_0^1 dx \left[\frac{y^2}{2} - xy \right]_{y=x}^{y=1} = \text{viene}$$

↑
prim. risp.
a x

↑
prim. risp.
a y

Osservazione



$$\iint_{T_1} (x-y) dx dy = \iint_{T_2} |x-y| dx dy$$

Esempio 4

$$\iint_A |y - x^2| dx dy =$$

$$y - x^2 \geq 0 \Leftrightarrow y \geq x^2$$

$$= \iint_{A^+} (y - x^2) dx dy + \iint_{A^-} (x^2 - y) dx dy$$

$$= \iint_{A^+} (y - x^2) dx dy - \iint_{A^-} (y - x^2) dx dy$$

$$= \int_0^1 dx \int_{x^2}^1 dy (y - x^2) - \int_0^1 dx \int_0^{x^2} dy (y - x^2) = \text{cont' -}$$

