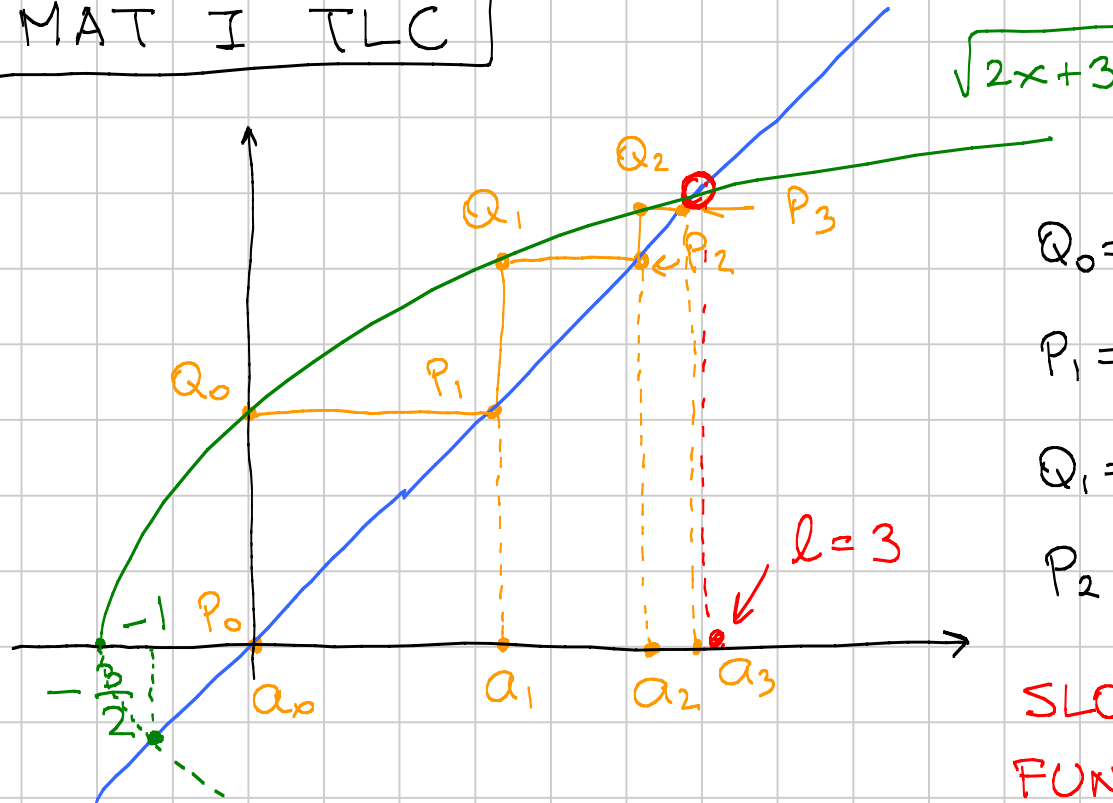




$$Q_0 = (a_0, f(a_0)) = (a_0, a_1)$$

$$P_1 = (a_1, a_1)$$

$$Q_1 = (a_1, f(a_1)) = (a_1, a_2)$$

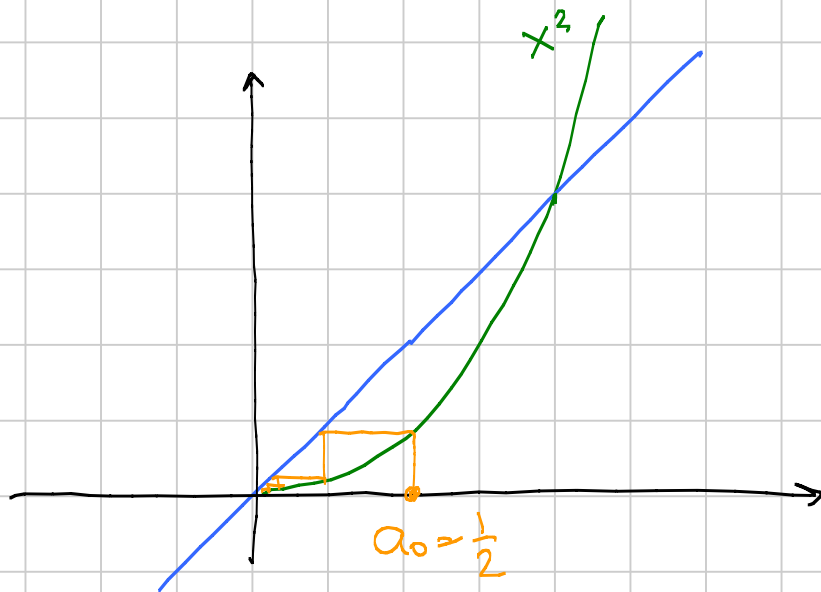
$$P_2 = (a_2, a_2)$$

SLOGAN: VERTICALE ALLA
FUNZIONE, ORIZZONTALE
ALLA BISETTRICE

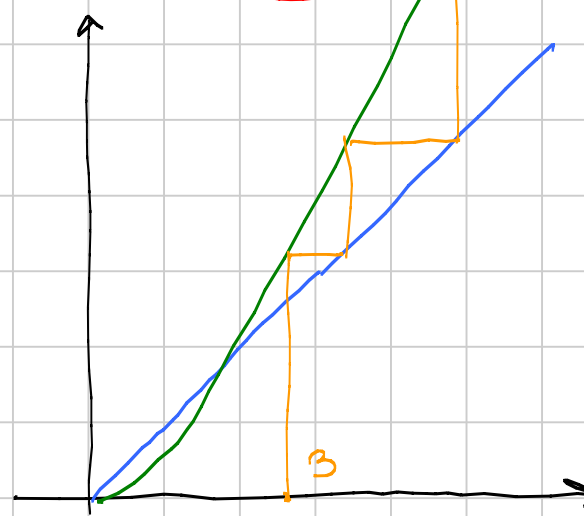
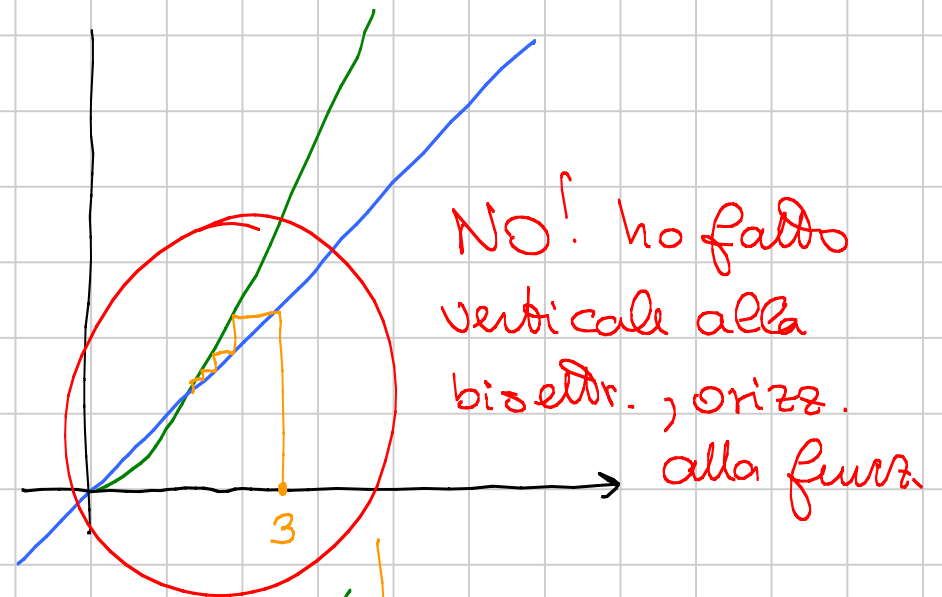
Per calcolare il punto di intersezione devo risolvere l'eq.

$$f(x) = x, \quad \text{cioè} \quad \sqrt{2x+3} = x$$

$$a_{n+1} = a_n^2, \quad a_0 = \frac{1}{2}$$



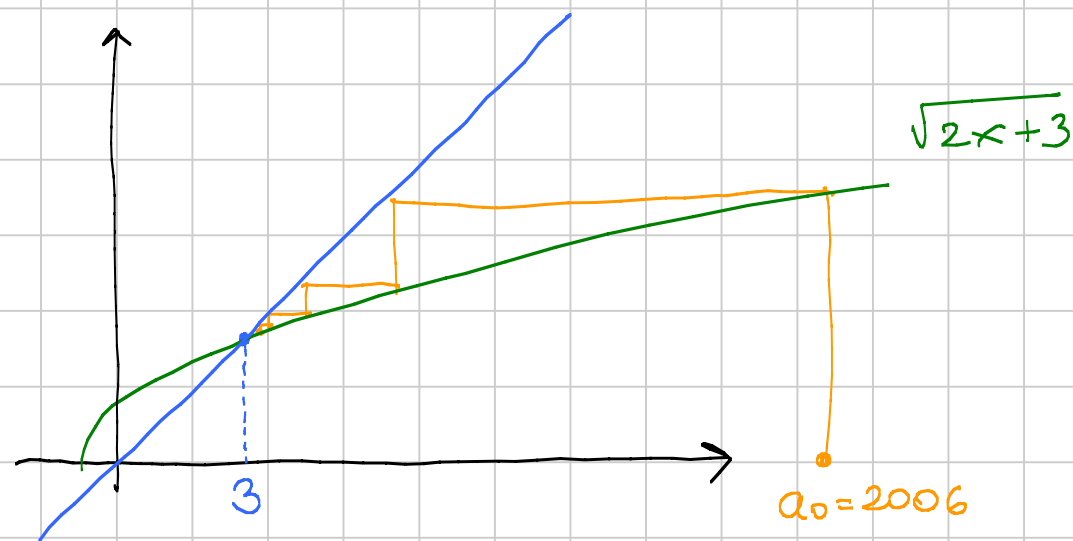
$$a_{n+1} = a_n^2, \quad a_0 = 3$$



Esempio 4

$$a_{n+1} = \sqrt{2a_n + 3}$$

$$a_0 = 2006$$



PIANO

(i) $a_n \geq 3 \quad \forall n \in \mathbb{N}$

(ii) $a_{n+1} \leq a_n \quad \forall n \in \mathbb{N}$

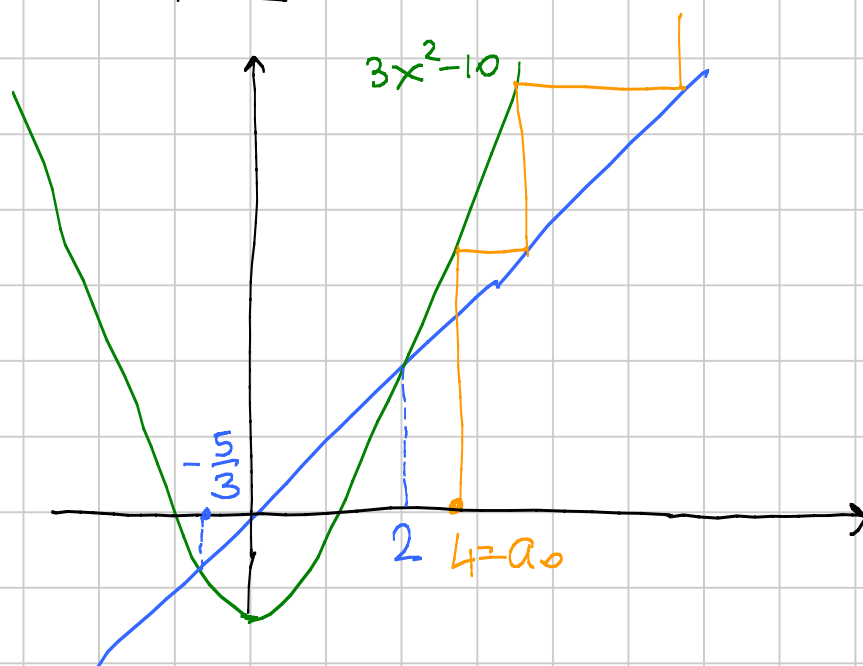
(iii) $a_n \rightarrow l \in \mathbb{R} \quad [(i) + (ii) + \text{teo. succ. monotone}]$

(iv) $l = 3 \quad [\dots \quad l = \sqrt{2l+3} \quad \dots]$

per induzione usando
la monotonia della
 $f(x) = \sqrt{2x+3}$

Esempio 5

$$a_{n+1} = 3a_n^2 - 10, \quad a_0 = 4$$



$$f(x) = 3x^2 - 10, \quad f(4) = 3 \cdot 16 - 10 = 38 > 4$$

PIANO

$$(i) \quad a_n \geq 4 \quad \forall n \in \mathbb{N}$$

$$(ii) \quad a_{n+1} \geq a_n \quad \forall n \in \mathbb{N}$$

$$(iii) \quad a_n \rightarrow l \in \mathbb{R} \cup \{+\infty\}$$

$$(iv) \quad l = +\infty$$

(i) : induzione

(ii) induzione + monotonia di $f(x)$ per $x \geq 0$

$$\boxed{n=0} \quad a_1 \geq a_0 \quad 38 \geq 4 \quad \text{O.K.}$$

$$\boxed{\text{P.I.}} \quad \text{Hp: } a_{n+1} \geq a_n \quad (\geq 4) \quad \text{Applico } f(x)$$

$$f(a_{n+1}) \geq f(a_n)$$

" "

$$a_{n+2} \geq a_{n+1} \leftarrow \text{Terzi del P.I.}$$

(iii) Teo. succ. monotone

(iv) Supponiamo per assurdo $l \in \mathbb{R}$

$$\begin{array}{ccc} a_{n+1} & = & 3a_n^2 - 10 \\ \downarrow & & \downarrow \\ l & = & 3l^2 - 10 \end{array}$$

$$\begin{aligned} 3l^2 - l - 10 &= 0 \\ l_{1,2} &= \frac{1 \pm \sqrt{1+120}}{6} = \frac{1 \pm 11}{6} \end{aligned}$$

$$l = \begin{pmatrix} 2 \\ -\frac{10}{6} \end{pmatrix} = -\frac{5}{3}$$

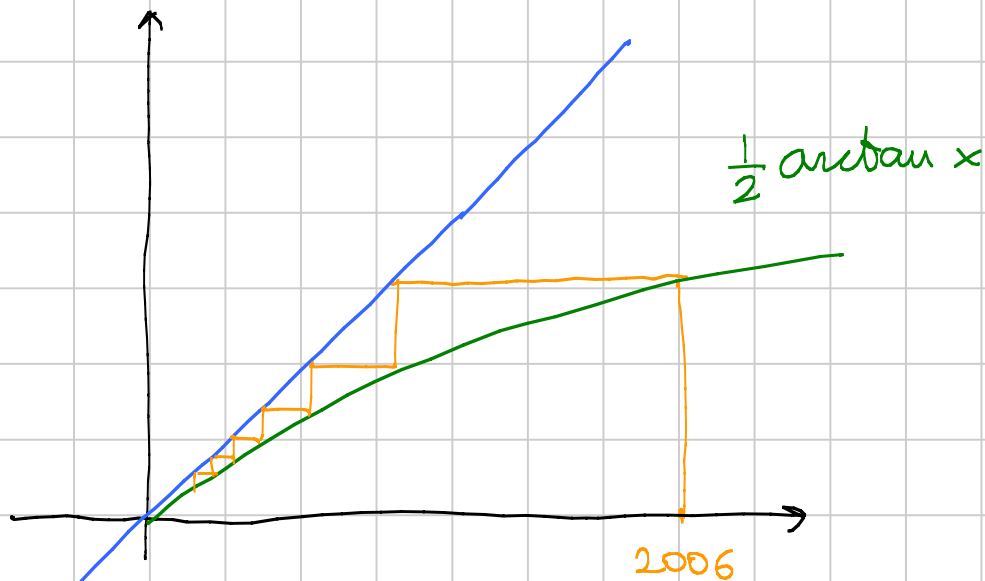
incompatibili con (i)

— 0 — 0 —

Esempio 6

$$a_{n+1} = \frac{1}{2} \arctan a_n$$

$$a_0 = 2006$$



Quando si vanno a disegnare x e $f(x)$ ci si pone il problema

di dove si intersecano

$$f(x) = x$$

nei restanti tratti chi sta sopra

$$f(x) > x$$

Quando va bene sono eq. e diseq. da percorso, quando va male occorre studiare la funzione $f(x) = x$.

Nel nostro caso per $x=0$ si incontrano, per $x>0$ si ha che

$$x > \arctan x > \frac{1}{2} \arctan x \Rightarrow x \text{ sta sopra}$$

↑
Disug. classica

- | | |
|--|---|
| <div style="border: 1px solid black; padding: 2px; display: inline-block;">PIANO</div> | <div style="display: inline-block; vertical-align: middle;"><div style="display: inline-block; vertical-align: middle;">(i) $a_n \geq 0 \quad \forall n \in \mathbb{N}$
(ii) $a_{n+1} \leq a_n \quad \forall n \in \mathbb{N}$
(iii) $a_n \rightarrow l \in \mathbb{R}$
(iv) $l = 0$</div><div style="display: inline-block; vertical-align: middle; font-size: 3em; margin: 0 10px;">}</div><div style="display: inline-block; vertical-align: middle;">solito inclus. +
monotonia di $f(x)$

[Solito]
[Eq. $l = \frac{1}{2} \arctan l$
$x = f(x)$]</div></div> |
|--|---|

Sappiamo alla fine che $a_n \rightarrow 0$.

$$\sum_{n=0}^{\infty} a_n \quad \text{che fa?}$$

- ① È a termini > 0 perché si dice per involu. (punto (i))
- ② Cond. nec. è verificata perché $a_n \rightarrow 0 \Rightarrow$ PUÒ CONVERG.

Criterio del rapporto:

$$\frac{a_{n+1}}{a_n} = \frac{\frac{1}{2} \operatorname{arctan} a_n}{a_n} = \frac{1}{2} \boxed{\frac{\operatorname{arctan} a_n}{a_n}} \xrightarrow{\text{limite notevole}} \frac{1}{2} < 1$$

$\Rightarrow$ la serie converge.

$$a_{n+1} = e^{-a_n}$$

$$a_0 = 0$$

