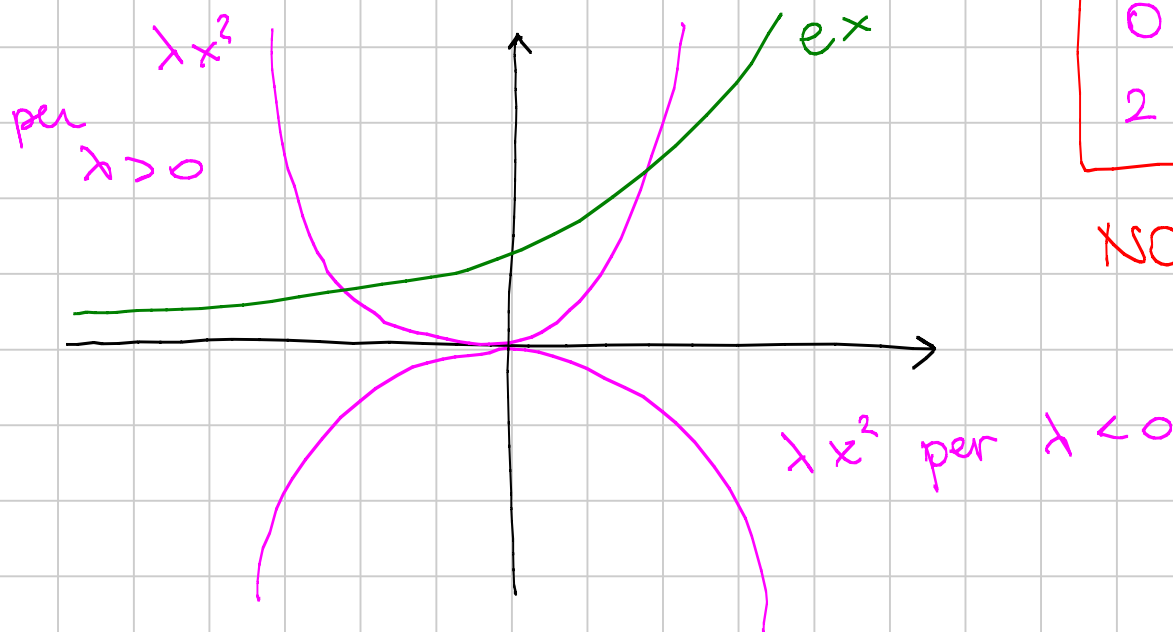


Dire al variare del param. $\lambda \in \mathbb{R}$ quante sono le solus. dell' eq.

$$e^x = \lambda x^2$$



0 solus. per $\lambda \leq 0$
2 solus. per $\lambda > 0$

NO!!! mai fidarsi a
sovrapporre 2
grafici

$$\boxed{\frac{e^x}{x^2}} = \lambda$$

"
f(x)

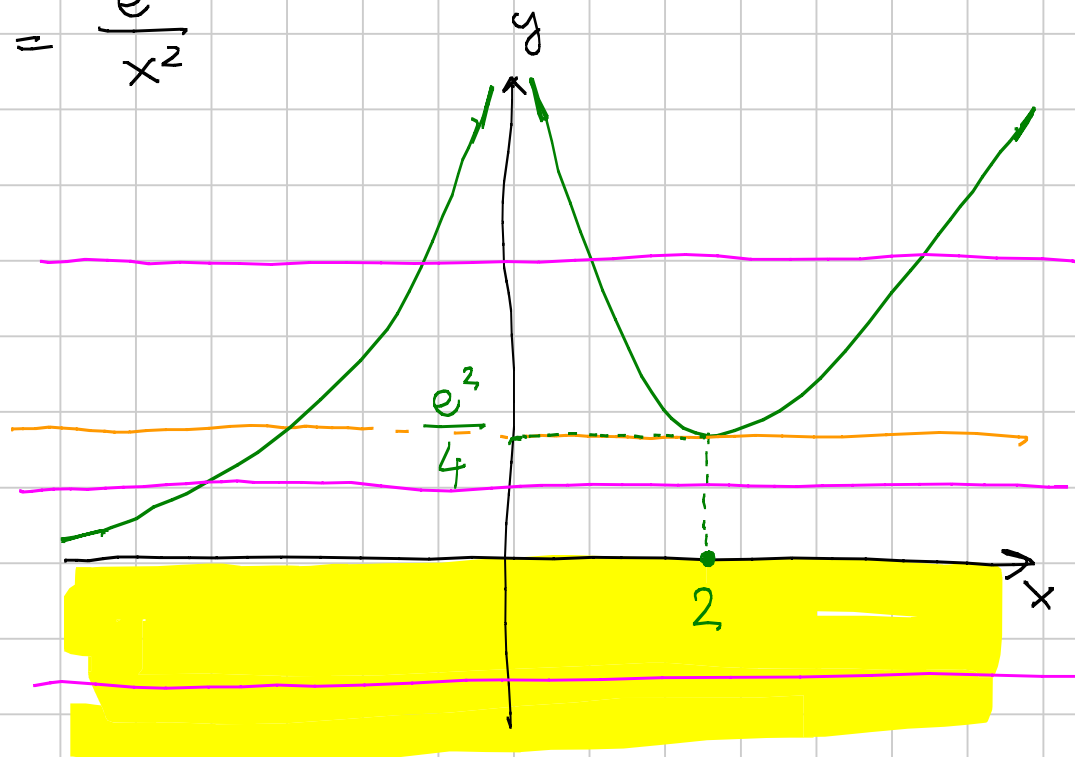
$$f(x) = \frac{e^x}{x^2}$$

Definita per
 $x \neq 0$

$$f'(x) = \frac{e^x x^2 - 2x e^x}{x^4}$$

$$= \frac{x e^x (x-2)}{x^4}$$

$$= \frac{e^x (x-2)}{x^3} = 0 \Leftrightarrow x = 2$$



[Per esercizio dimostrare che
 $f'(x) > 0$ per $x < 0$ e $x > 2$
 $f'(x) < 0$ per $0 < x < 2$]

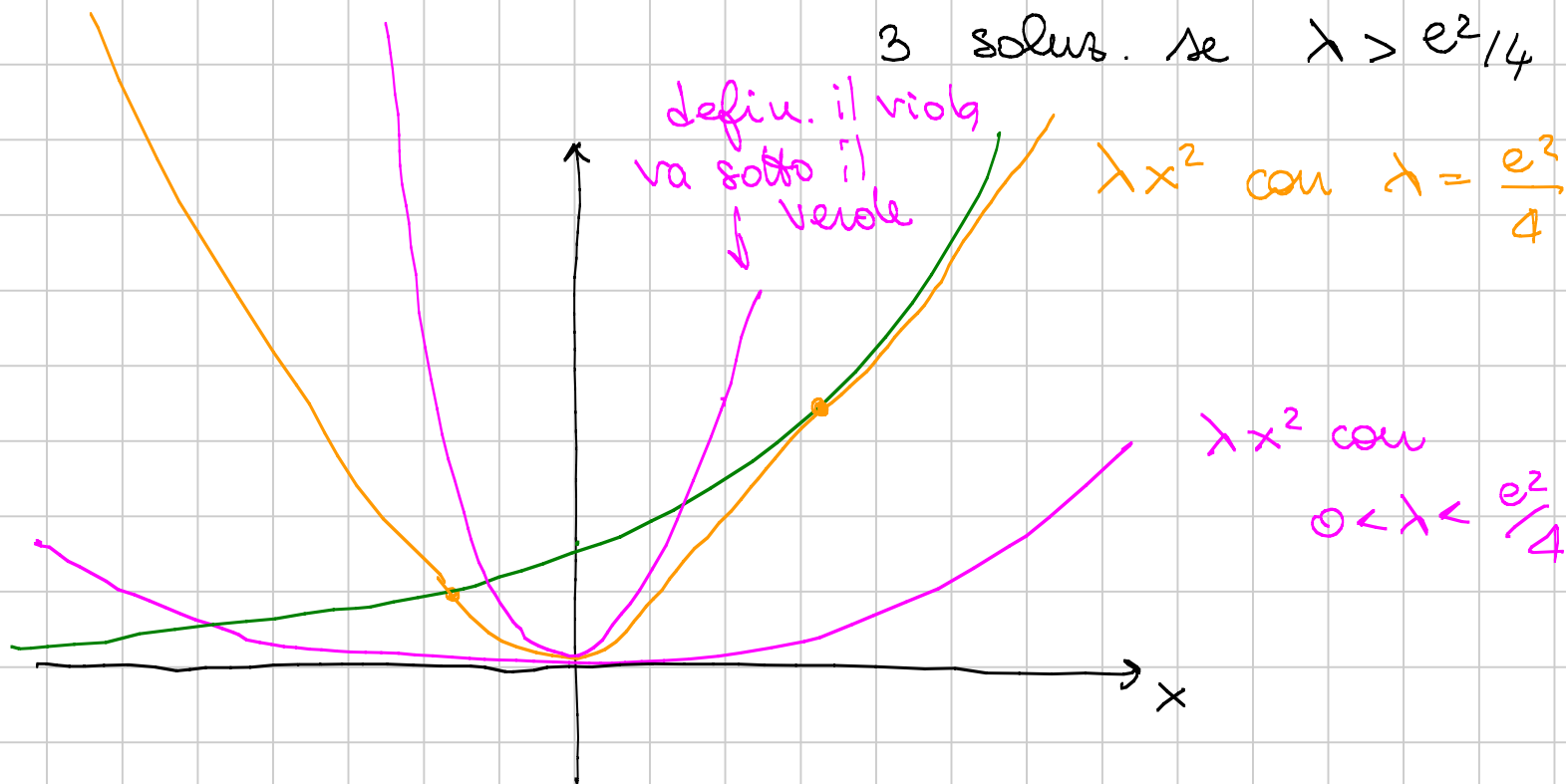
Quindi $f(x) = \lambda$ ha

0 solus. se $\lambda \leq 0$

1 solus. se $0 < \lambda < e^2/4$

2 solus. se $\lambda = e^2/4$

3 solus. se $\lambda > e^2/4$



Risolvere la diseq,

$$\boxed{2x - \sin x} \geq 0$$

$f(x)$



$$f'(x) = 2 - \cos x \geq 1 \quad \forall x \in \mathbb{R} \Rightarrow f \text{ strett. monot. cresc.}$$

$$f(0) = 0 \Rightarrow \begin{array}{l} f(x) > 0 \text{ per ogni } x > 0 \\ f(x) < 0 \text{ per ogni } x < 0 \end{array}$$

Risolvere la diseq,

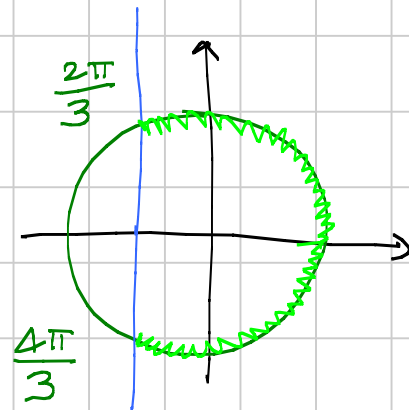
$$\boxed{x + 2\sin x} \geq 0$$

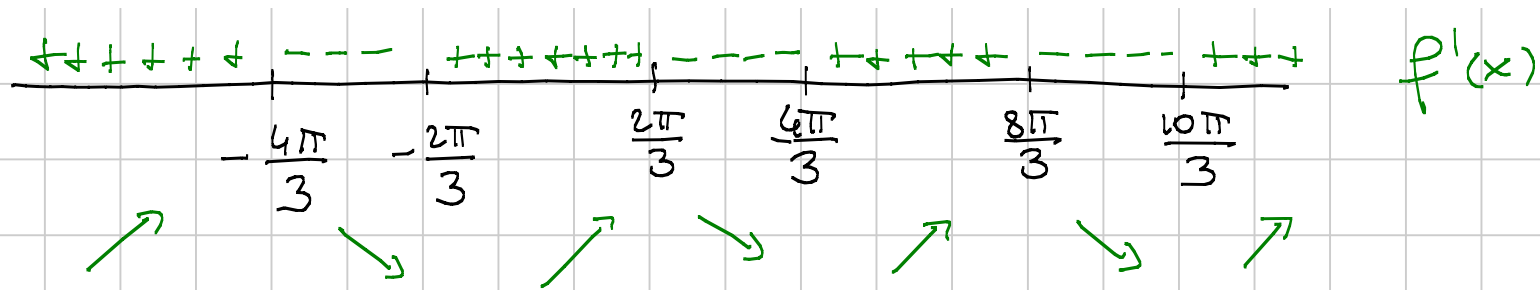
$f(x)$

$$f'(x) = 1 + 2\cos x$$

studio il segno

$$1 + 2\cos x \geq 0 \Leftrightarrow \cos x \geq -\frac{1}{2}$$





per $x < 0$ è
analogo
(è dispari)

Sostituisco $x = \frac{4\pi}{3}$

$$f\left(\frac{10\pi}{3}\right) = \frac{10\pi}{3} - 2 \sin\left(\frac{10\pi}{3}\right)$$

$$= \frac{10\pi}{3} - \sqrt{3} \quad \leftarrow \text{Tolgo sempre } \sqrt{3}$$

$$f\left(\frac{4\pi}{3}\right) = \frac{4\pi}{3} + 2 \sin\left(\frac{4\pi}{3}\right)$$

$$= \frac{4\pi}{3} + 2\left(-\frac{\sqrt{3}}{2}\right)$$

$$= \frac{4\pi}{3} - \sqrt{3} > 0$$



$$x + 2\sin x \geq 0 \Leftrightarrow x \geq 0$$

$$x + 2\sin x = 0 \Leftrightarrow x = 0$$

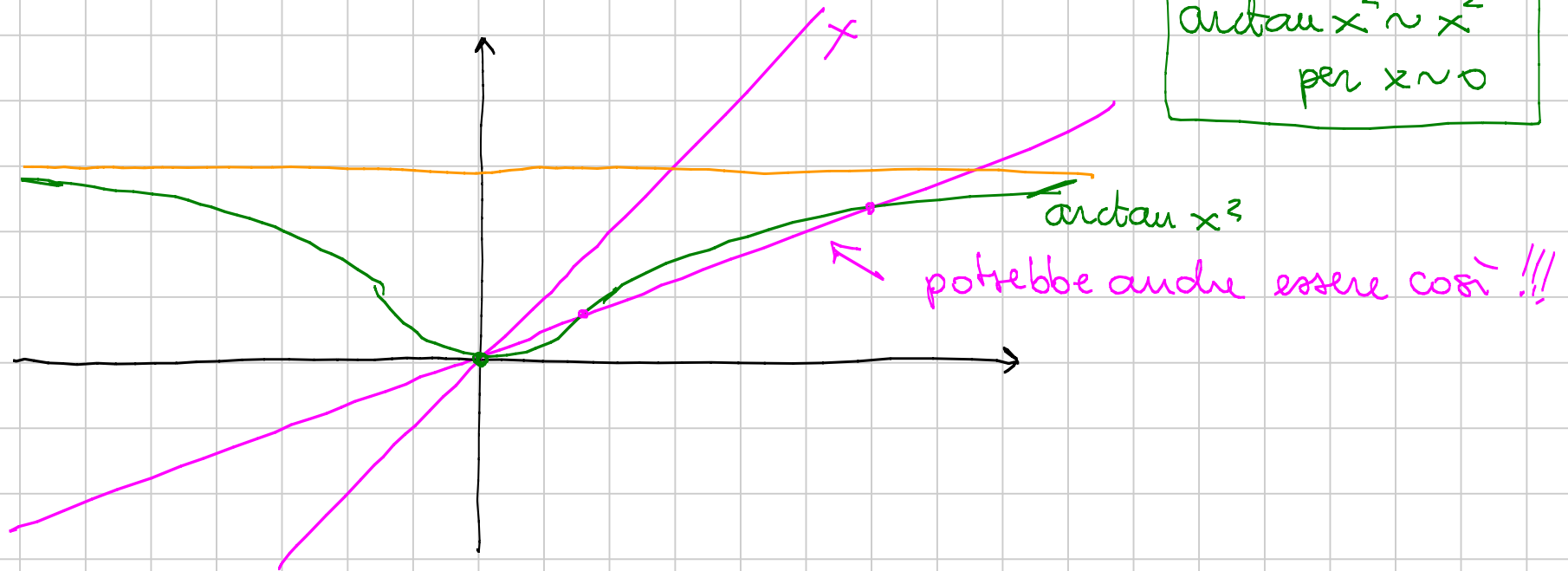
— 0 — 0 —

Risolvere la diseq.

$$\arctan x^2 \leq x$$

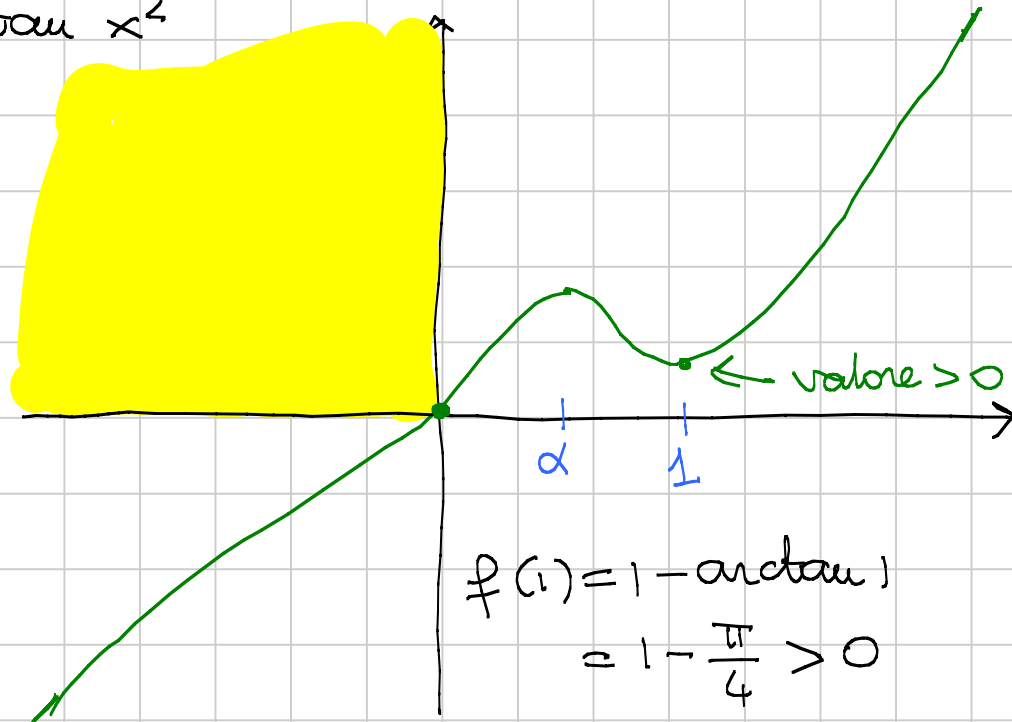
STUDIO LOCALE

$$\arctan x^2 \sim x^2 \text{ per } x \sim 0$$



Studio $f(x) = x - \arctan x^2$

$$f'(x) = 1 - \frac{2x}{1+x^4}$$
$$= \frac{1+x^4-2x}{1+x^4}$$



$$f'(x) \geq 0 \Leftrightarrow$$

$$\boxed{x^4 - 2x + 1} \geq 0 \quad \text{per } x \leq 0$$

$g(x)$

$$\begin{matrix} x^4 & -2x & +1 \\ \geq 0 & \geq 0 & \end{matrix} \geq 1 \Rightarrow \text{monotona} \\ \text{crescente}$$

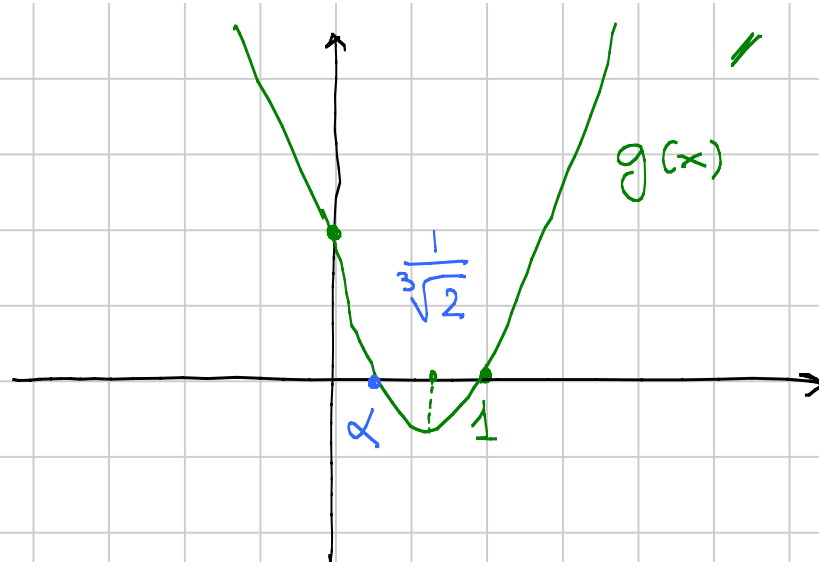
Studio $g(x) = x^4 - 2x + 1$

$$g'(x) = 4x^3 - 2$$

$$g'(1) = 2$$

$$g''(x) = 12x^2 \geq 0 \text{ sempre}$$

⇓
Convessa



$$g'(x) \geq 0 \Leftrightarrow 4x^3 - 2 \geq 0 \Leftrightarrow$$

$$2x^3 \geq 1 \Leftrightarrow x^3 \geq \frac{1}{2} \Leftrightarrow x \geq \sqrt[3]{\frac{1}{2}}$$

Cosa posso dire del segno di $g(x)$?

$$g(x) > 0$$

per $x > 1$ e per $x < \alpha$ (so che $\alpha \in (0, \sqrt[3]{\frac{1}{2}})$)

$$g(x) < 0$$

per $\alpha < x < 1$

$$g(x) = 0$$

per $x = \alpha$ e $x = 1$

Ora so che $f'(x)$ è così



Conclusione $f(x) \geq 0 \Leftrightarrow x \geq 0$ e quindi

$$x \geq 0 \text{ and } x^2 \Leftrightarrow x \geq 0$$

— 0 — 0 —

$$\max \{ x \in \mathbb{R} : x e^{-x} \leq 0 \} = 0$$

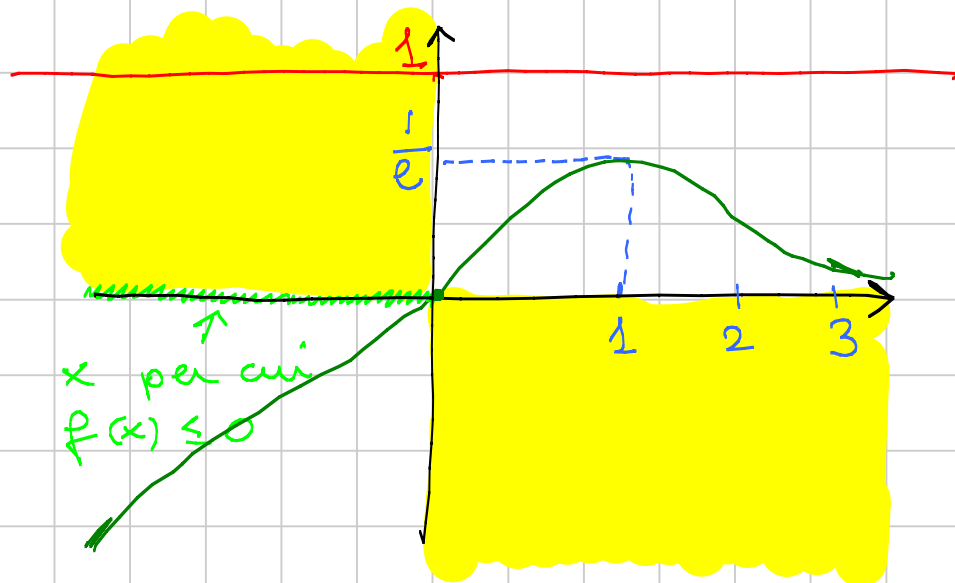
MONDO X

RISOLVI LA DISEQUAZIONE
e prendi il max degli
x ottenuti

$$\max \{ x e^{-x} : x \in \mathbb{R} \} = \frac{1}{e}$$

MONDO y

TROVA il max (valore) della funzione al variare di $x \in \mathbb{R}$



$$f'(x) = e^{-x} - x e^{-x}$$

$$= e^{-x} (1-x) = 0 \Rightarrow x=1$$

[Controllare il segno di $f'(x)$]

$$\max \{ x e^{-x} : x \in [2, 3] \} = f(2) = \frac{2}{e^2}$$

MONDO y

La sol. della diseq. è
↑ tutto $\mathbb{R}$, quindi

$$\sup \{ x \in \mathbb{R} : x e^{-x} < 1 \} = +\infty$$

$$\min \{ x e^{-x} : x \in [2, 3] \} = f(3) = \frac{3}{e^3}$$

MONDO x

Risolvi la diseq.

$f(x) < 1$ e prendi il sup delle
insieme soluz.