

FUNZIONI IPERBOLICHE

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

↑
seno iperbolico

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

↑
coseno iperbolico

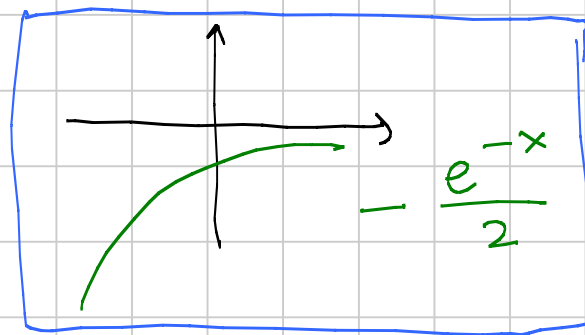
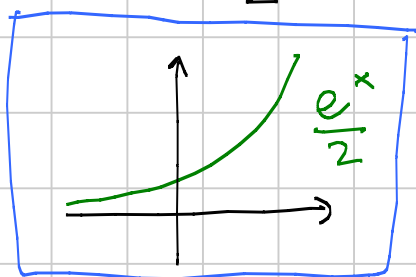
$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

SIMMETRIE ?

$$\cosh(-x) = \frac{e^{-x} + e^x}{2} = \cosh x \quad \text{PARI}$$

$$\sinh(-x) = \frac{e^{-x} - e^x}{2} = -\sinh x \quad \text{DISPARI}$$

$\sinh x$ è la somma di $\frac{e^x}{2}$ e $-\frac{e^{-x}}{2}$

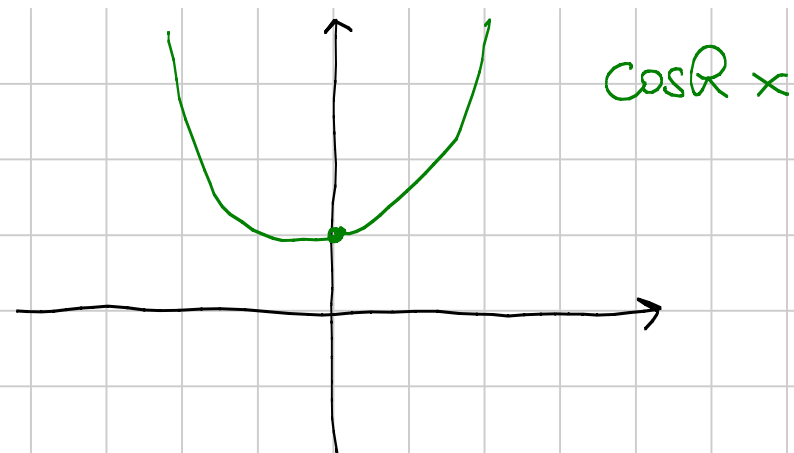
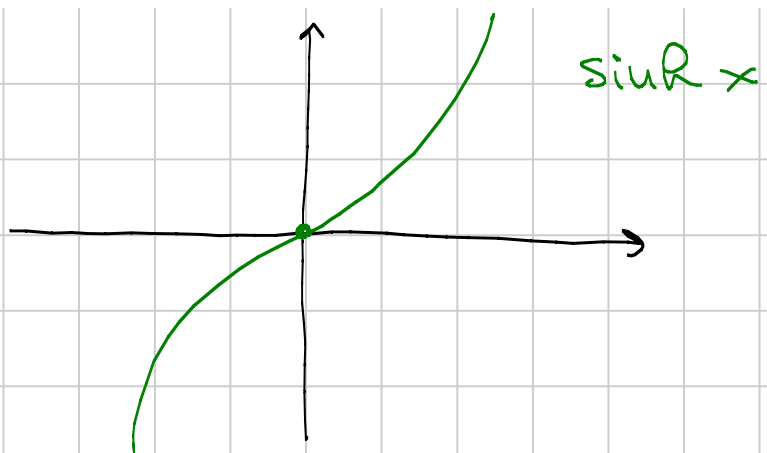


$\sinh x$, essendo somma di 2 funzioni strett. cresc., sarà a sua volta strett. cresc.

$$\lim_{x \rightarrow +\infty} \sinh x = \lim_{x \rightarrow +\infty} \frac{e^x - e^{-x}}{2} = +\infty$$

$$\sinh 0 = \frac{e^0 - e^0}{2} = 0$$

$$\lim_{x \rightarrow +\infty} \cosh x = \lim_{x \rightarrow +\infty} \frac{e^x + e^{-x}}{2} = +\infty$$



$$\cosh 0 = \frac{e^0 + e^0}{2} = \frac{1+1}{2} = 1$$



$$(\sinh x)' = \left(\frac{e^x - e^{-x}}{2} \right)' = \frac{e^x + e^{-x}}{2} = \cosh x$$

$$(\cosh x)' = \left(\frac{e^x + e^{-x}}{2} \right)' = \frac{e^x - e^{-x}}{2} = \sinh x$$

$$\begin{aligned} \sinh(2x) &= \frac{e^{2x} - e^{-2x}}{2} = \frac{(e^x + e^{-x})(e^x - e^{-x})}{2} \\ &= 2 \cdot \frac{e^x + e^{-x}}{2} \cdot \frac{e^x - e^{-x}}{2} \end{aligned}$$

$$\sinh(2x) = 2 \sinh x \cdot \cosh x$$

Per esercizio: trovare formule per $\cosh(2x)$

Sviluppi di Taylor.

$$\cos x = \frac{1}{2} (e^x + e^{-x})$$

$$= \frac{1}{2} \left(\begin{array}{l} 1 + \cancel{x} + \frac{x^2}{2!} + \cancel{\frac{x^3}{3!}} + \frac{x^4}{4!} + \cancel{\frac{x^5}{5!}} + \dots \\ + 1 - \cancel{x} + \frac{x^2}{2!} - \cancel{\frac{x^3}{3!}} + \frac{x^4}{4!} - \cancel{\frac{x^5}{5!}} + \dots \end{array} \right) \quad \begin{array}{l} e^x \\ e^{-x} \end{array}$$

$$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$$

* termini con esp. pari
nello sviluppo di e^x

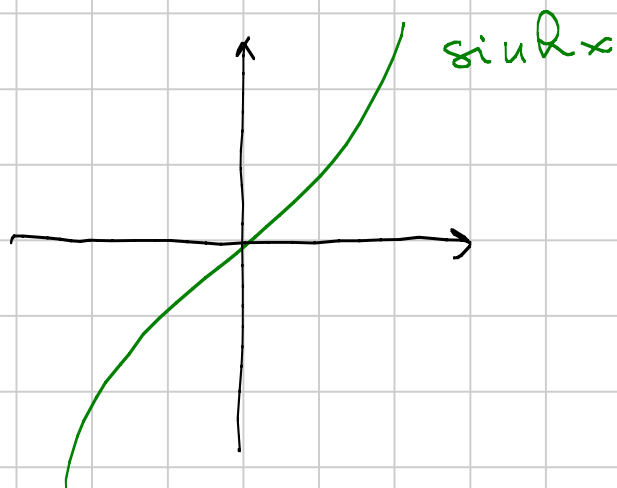
* stesso sviluppo di $\sin x$,
solo con tutti +

$\sin x =$ se ne vanno i pari e restano solo i dispari

$$= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$$

N.B.

$$\sinh x + \cosh x = \frac{e^x - e^{-x}}{2} + \frac{e^x + e^{-x}}{2} = e^x$$

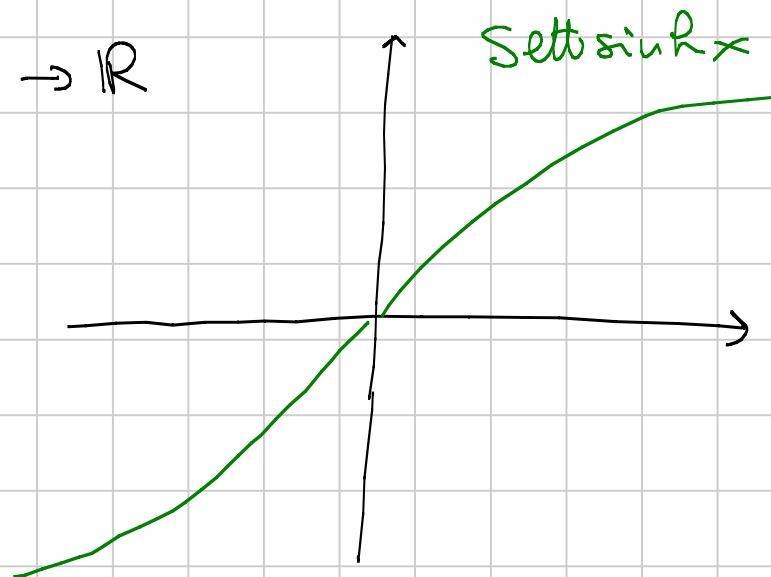


$f(x) = \sinh x$ $f: \mathbb{R} \rightarrow \mathbb{R}$
è iniettiva e surgettiva, dunque
invertibile.

La sua inversa è una

$$g: \mathbb{R} \rightarrow \mathbb{R}$$

detta $g(x) = \operatorname{Settsinh} x$



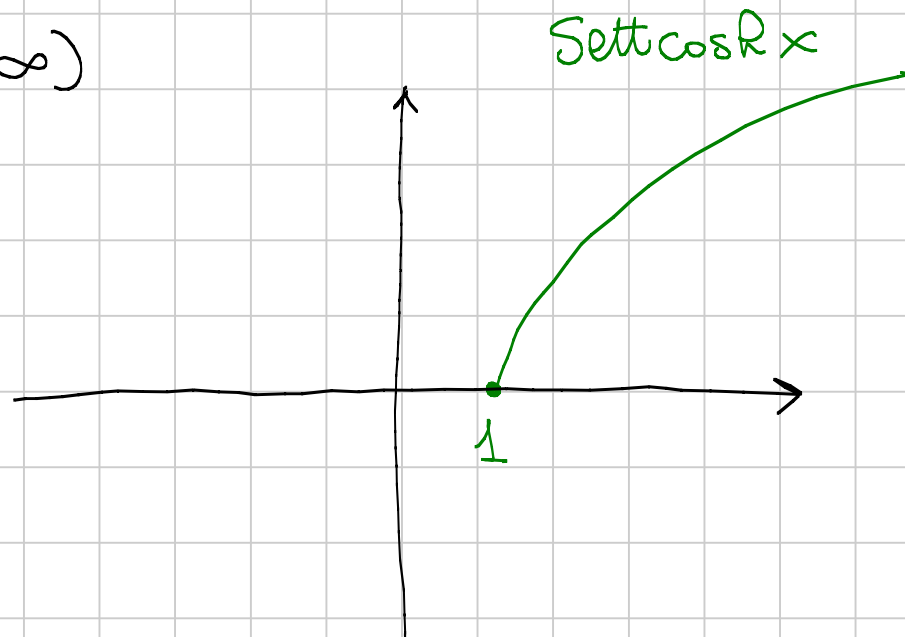
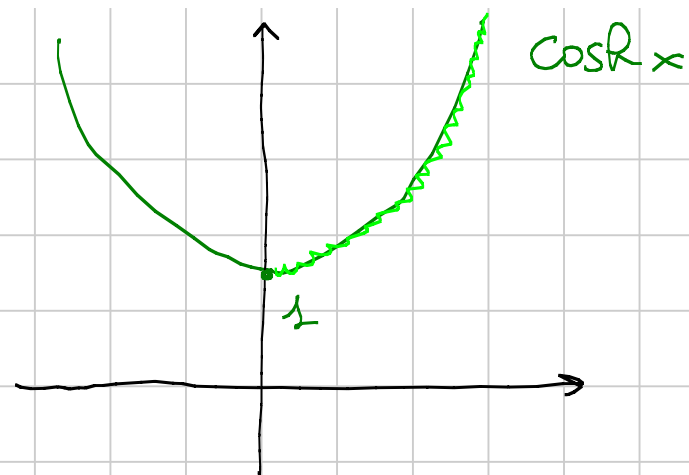
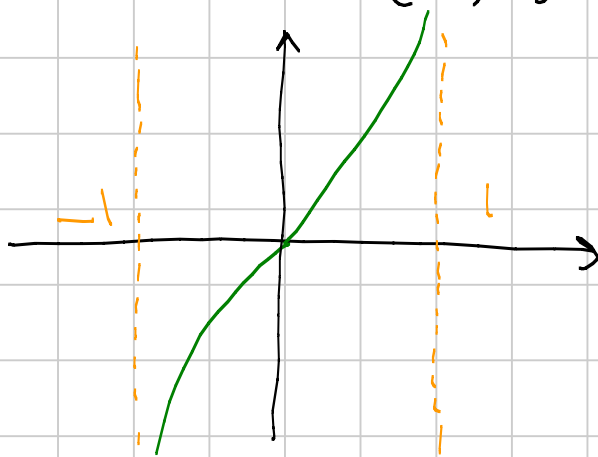
$$f(x) = \cos R x$$

$$f: [0, +\infty) \rightarrow [1, +\infty)$$

La sua inversa, $\text{Sett} \cos R x$,
è una funzione

$$g: [1, +\infty) \rightarrow [0, +\infty)$$

$$\text{Sett} \tan R: (-1, 1) \rightarrow \mathbb{R}$$



Ricaviamo una formula per $\text{Settsinh } x$ (per es. $\text{Settcosh } x$)

$$\sinh x = y$$

Provo a ricavare x in funzione di y

$$\frac{e^x - e^{-x}}{2} = y \quad ; \quad e^x - e^{-x} = 2y \quad \text{Pongo } z = e^x$$

$$z - \frac{1}{z} = 2y \quad ; \quad z^2 - 1 = 2yz \quad ;$$

$$z^2 - 2yz - 1 = 0$$

Ricavo z (in funzione di y)

$$z_{1,2} = y \pm \sqrt{y^2 + 1} \quad \begin{array}{l} \nearrow y + \sqrt{y^2 + 1} \\ \searrow y - \sqrt{y^2 + 1} \end{array}$$

2 possibili
valori per z

$$e^x = y + \sqrt{y^2 + 1}$$

$$\leadsto x = \log(y + \sqrt{y^2 + 1}) = \text{Settsinh } y$$

FORMULA

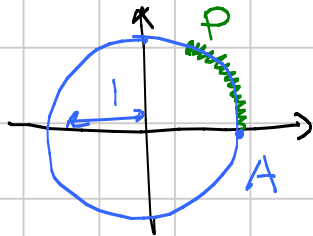
$$e^x = \boxed{y - \sqrt{y^2 + 1}} \rightsquigarrow \text{NULLA}$$

$$< 0 \text{ perché} \\ y - \sqrt{y^2 + 1} < 0$$

— 0 — 0 —

GEOMETRICAMENTE

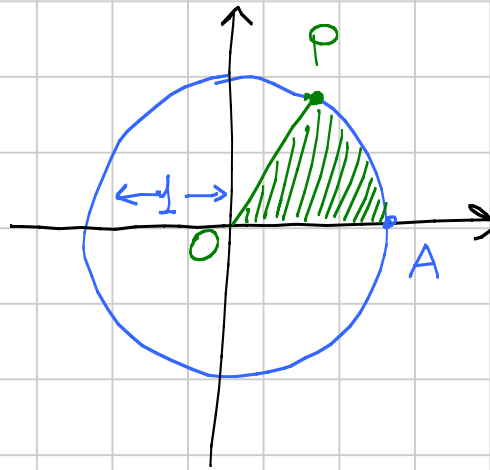
Trigo.
classica



arco lungo x , individuo P

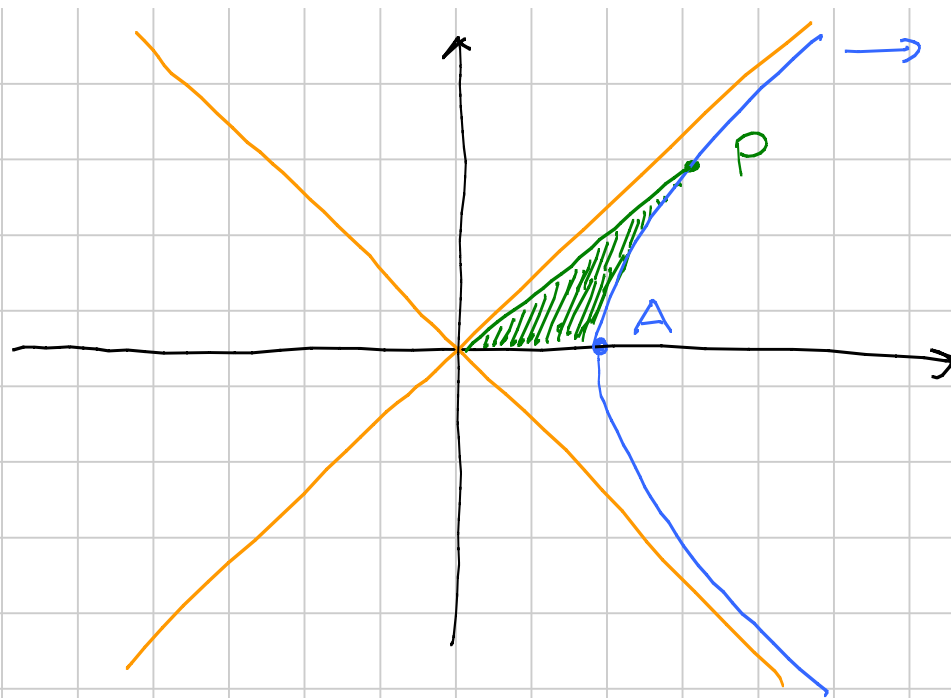
$$P = (\cos x, \sin x)$$

Trigo.
alternativa



Per quanto finché l'area del
settore è $x/2$

$$P = (\cos x, \sin x)$$

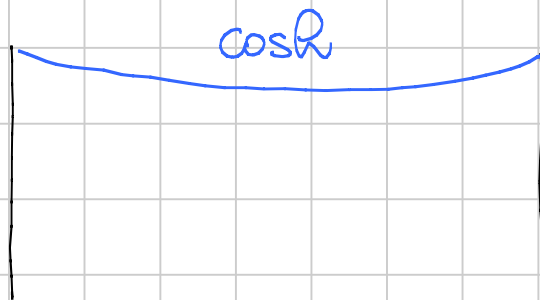


→ iperbole di equazione

$$x^2 - y^2 = 1$$

Percorro finché l'area
del settore è $\frac{x}{2}$

$$P = (\cosh x, \sinh x)$$



PALI

