

MAT 1 TLC

ORA 23

$$o(h g'(x_0) + o(h)) \Rightarrow o(h)$$

$$(h g'(x_0) + o(h)) \omega_1(h) =$$

$$= (h g'(x_0) + h \omega_2(h)) \omega_1(h)$$

$$= h \underbrace{(g'(x_0) + \omega_2(h)) \omega_1(h)}_{\omega_3(h)} = o(h)$$

$$\omega_3(h) \rightarrow 0 \text{ per } h \rightarrow 0$$

ESERCIZI

$$(1) \quad \lim_{x \rightarrow -\infty} \sqrt{x^2 + 3x} + x \quad [+\infty - \infty]$$

Pongo $y = -x$. Quando $x \rightarrow -\infty$, ho che $y \rightarrow +\infty$

$$= \lim_{y \rightarrow +\infty} \sqrt{y^2 - 3y} - y$$

$$\left(\overset{A}{\sqrt{y^2 - 3y}} - \overset{B}{y} \right) \frac{\overset{A}{\sqrt{y^2 - 3y}} + \overset{B}{y}}{\overset{A}{\sqrt{y^2 - 3y}} + \overset{B}{y}} = \frac{\overset{A^2}{\cancel{y^2}} - 3y - \overset{B^2}{\cancel{y^2}}}{\sqrt{y^2 - 3y} + y} =$$

$$= \frac{-3y}{\sqrt{y^2 - 3y} + y} = \frac{-3\cancel{y}}{\cancel{y} \left[\sqrt{1 - \frac{3}{y}} + 1 \right]} \rightarrow -\frac{3}{2}$$

(2)

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \cos x}{x^2} = 1$$

$$\frac{1-1}{0} = \frac{0}{0}$$

$$\frac{\sqrt{1+x^2} - \cos x + 1 - 1}{x^2} = \frac{1 - \cos x}{x^2} + \frac{\sqrt{1+x^2} - 1}{x^2} \rightarrow 1$$

$$\downarrow \frac{1}{2}$$

$$\downarrow \frac{1}{2}$$

$$\frac{\sqrt{1+x^2} - 1}{x^2} \cdot \frac{\sqrt{1+x^2} + 1}{\sqrt{1+x^2} + 1} = \frac{\cancel{1+x^2} - 1}{\cancel{x^2} (\sqrt{1+x^2} + 1)} = \frac{1}{\sqrt{1+x^2} + 1} \rightarrow \frac{1}{2}$$

③ DA TABELLINA

$$\lim_{x \rightarrow 0^+} x \cdot \log x = [0 \cdot (-\infty)]$$

Pongo $y = \log x$. $\nearrow x = e^y$ Quando $x \rightarrow 0^+$, ho che $y \rightarrow -\infty$

$$= \lim_{y \rightarrow -\infty} e^y \cdot y$$

Pongo $z = -y$. Quando $y \rightarrow -\infty$,
ho che $z \rightarrow +\infty$

$$= \lim_{z \rightarrow +\infty} e^{-z} \cdot (-z) = \lim_{z \rightarrow +\infty} -\frac{z}{e^z} = 0$$

$\uparrow$
potenza / esponente,

$\lim_{x \rightarrow 0^+} x \cdot \log x = 0$. Non cambia anche se metto
 $x^a \cdot \log x$ $a > 0$

$$\textcircled{4} \quad \lim_{x \rightarrow 0^+} x^{\sin x} \quad [0^0] = e^0 = 1,$$

$$x^{\sin x} = e^{\sin x \cdot \log x}$$

Basta fare il lim. dell'esponente,

$$\sin x \cdot \log x = \boxed{\frac{\sin x}{x}} \cdot \boxed{x \log x} \rightarrow 0$$

$\downarrow$ $\downarrow$
 1 0

$$\textcircled{5} \quad \lim_{x \rightarrow 0^+} x^{\arctan \sqrt{x}} = \lim_{x \rightarrow 0^+} e^{\arctan \sqrt{x} \cdot \log x} = e^0 = 1$$

$$\arctan \sqrt{x} \cdot \log x = \boxed{\frac{\arctan \sqrt{x}}{\sqrt{x}}} \cdot \boxed{\sqrt{x} \log x} \rightarrow 0$$

↓
1

↓
0
a = 1/2

$$(6) \lim_{n \rightarrow +\infty} n^2 \arctan \frac{6n}{3n^3 + 5} = 2 \quad \left[+\infty \cdot \overset{0}{\arctan(0)} \right]$$

$$n^2 \arctan \frac{6n}{3n^3 + 5} = \boxed{n^2} \cdot \boxed{\frac{\arctan \frac{6n}{3n^3 + 5}}{\frac{6n}{3n^3 + 5}}} \cdot \boxed{\frac{6n}{3n^3 + 5}}$$

$$\frac{6n^3}{3n^3 + 5} \rightarrow 2$$

Pongo $x = \frac{6n}{3n^3 + 5}$
 $\dots x \rightarrow 0$

↓
1

$$\textcircled{7} \quad \lim_{n \rightarrow +\infty} n \left(\sqrt[n]{37} - 1 \right) \quad +\infty (1-1)$$

$$n \left(\sqrt[n]{37} - 1 \right) = n \left(37^{\frac{1}{n}} - 1 \right) = n \left(e^{\frac{1}{n} \log 37} - 1 \right)$$

$$= \frac{e^{\frac{1}{n} \log 37} - 1}{\frac{1}{n} \log 37} \cdot \boxed{\log 37} = \log 37$$

$$x = \frac{\log 37}{n} \rightarrow 0$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$$

$$\rightarrow 1$$

$$\log 37$$

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$$\lim_{n \rightarrow +\infty} \sqrt{n} \left(\sqrt[3]{n} - 1 \right) = 0$$

$$[+\infty \cdot (1-1)]$$

$$\sqrt{n} \left(n^{\frac{1}{3}} - 1 \right) = \sqrt{n} \left(e^{\frac{1}{3} \log n} - 1 \right)$$

$$x = \frac{\log n}{3} \rightarrow 0$$

$$= \frac{e^{\frac{1}{3} \log n} - 1}{\frac{\log n}{n}} \cdot \frac{\log n}{n} \sqrt{n} \rightarrow 0$$

$\downarrow$
 1

$\downarrow$
 0

$$\textcircled{9} \quad \lim_{x \rightarrow 0} (\cos x)^{\frac{1}{x^2}} \quad [1^\infty]$$

$$(\cos x)^{\frac{1}{x^2}} = e^{\frac{1}{x^2} \log(\cos x)} \rightarrow e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

$$\frac{\log(\cos x)}{x^2} = \frac{\log(1 + \cos x - 1)}{x^2} =$$

$$= \boxed{\frac{\cos x - 1}{x^2}} \cdot \boxed{\frac{\log(1 + \cos x - 1)}{\cos x - 1}} \rightarrow -\frac{1}{2}$$

$$\downarrow$$

$$-\frac{1}{2}$$

$$\parallel$$

$$\lim_{y \rightarrow 0} \frac{\log(1+y)}{y} = 1$$

Pongo $y = \cos x - 1$.

Quando $x \rightarrow 0$, ho che $y \rightarrow 0$