

MAT 1 TLC

ORA 22

$$R^2 f'(x_0) g'(x_0) \stackrel{?}{=} o(R)$$

In generale

$$kR^2 = R \cdot \underbrace{kR}_{w(R)} \quad \lim_{R \rightarrow 0} w(R) = \infty$$
$$= o(R)$$

$$R \cdot o(R) = o(R) \quad \text{per } R \rightarrow 0$$

$$R \cdot R w(R) = R \underbrace{(R w(R))}_{w_1(R) \rightarrow 0} = R w_1(R)$$

Derivate delle Funzioni elementari

$$\textcircled{1} \quad f(x) = x^2 \quad x_0 \in \mathbb{R}$$

$$\lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h} = \lim_{h \rightarrow 0} \frac{(x_0+h)^2 - x_0^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x_0^2} + 2x_0h + h^2 - \cancel{x_0^2}}{h} = \lim_{h \rightarrow 0} (2x_0 + h) = 2x_0$$

In alternativa;

$$f(x_0+h) = (x_0+h)^2 = x_0^2 + 2x_0 \cdot h + h^2$$

$$= f(x_0) + \boxed{2x_0} \cdot h + o(h)$$

$f'(x_0)$

1.5) $f(x) = x^k$

$f'(x_0) = k x_0^{k-1}$ (si dim. per induzione usando la deriv. del prodotto).

② $f(x) = e^x$

$$\lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h} = \lim_{h \rightarrow 0} \frac{e^{x_0+h} - e^{x_0}}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{e^{x_0} \cdot e^h - e^{x_0}}{h} = \lim_{h \rightarrow 0} e^{x_0} \left(\frac{e^h - 1}{h} \right) = e^{x_0}$$

$$e^{a+b} = e^a \cdot e^b$$

↓ limite
1 notevole

$$f(x_0+h) = e^{x_0+h} = e^{x_0} \cdot e^h =$$

[sviluppiui]

$$= e^{x_0} (1 + h + o(h))$$

$$= e^{x_0} + e^{x_0} \cdot h + e^{x_0} \cdot o(h)$$

$$= f(x_0) + \boxed{e^{x_0}} \cdot h + o(h)$$

$f'(x_0)$

③ $f(x) = \sin x$

$$\lim_{h \rightarrow 0} \frac{\sin(x_0 + h) - \sin x_0}{h}$$

1° modo: usare formule
per $\sin \alpha - \sin \beta$ con
 $\alpha = x_0 + h$ e $\beta = x_0$

2° modo: usare formule
per $\sin(x_0 + h)$

$$= \lim_{h \rightarrow 0} \frac{\sin x_0 \cdot \cos h + \cos x_0 \cdot \sin h - \sin x_0}{h}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \left(\boxed{\cos x_0} \cdot \boxed{\frac{\sin h}{h}} + \boxed{\sin x_0} \cdot \boxed{\frac{\cos h - 1}{h}} \right) = \cos x_0 \\
 &\quad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \\
 &\cos x_0 \qquad 1 \qquad + \sin x_0 \cdot 0
 \end{aligned}$$

$$\begin{aligned}
 &\qquad \qquad \qquad \downarrow \\
 &\qquad \qquad \qquad \boxed{\frac{\cos h - 1}{h^2}} \cdot h \\
 &\qquad \qquad \qquad \downarrow \qquad \qquad \downarrow \\
 &\qquad \qquad \qquad -\frac{1}{2} \qquad 0
 \end{aligned}$$

$$\begin{aligned}
 f(x_0 + h) &= \sin(x_0 + h) = \sin x_0 \cdot \cos h + \cos x_0 \cdot \sin h \\
 &= \sin x_0 (1 + o(h)) + \cos x_0 (h + o(h)) \\
 &= \sin x_0 + \boxed{\sin x_0 \cdot o(h)} + \cos x_0 \cdot h + \boxed{\cos x_0 \cdot o(h)} \\
 &\qquad \qquad \qquad f'(x_0) \\
 &= f(x_0) + h \cdot \boxed{\cos x_0} + o(h)
 \end{aligned}$$

3.5 Da fare : derivata di $\cos x = f(x) \leadsto f'(x_0) = -\sin x_0$

4 $f(x) = \log x$

$$\lim_{h \rightarrow 0} \frac{\log(x_0 + h) - \log(x_0)}{h} = \left[\log a - \log b = \log \frac{a}{b} \right]$$

$$\lim_{h \rightarrow 0} \frac{\log\left(\frac{x_0 + h}{x_0}\right)}{h} = \lim_{h \rightarrow 0} \frac{\log\left(1 + \frac{h}{x_0}\right)}{\frac{h}{x_0} \cdot x_0} = \frac{1}{x_0}$$

se pongo $y = \frac{h}{x_0}$ diventa

$$\lim_{y \rightarrow 0} \frac{\log(1+y)}{y} = 1$$

$$(f \pm g)' = f' \pm g'$$

$$(\lambda f)' = \lambda f'$$

$$\left(\frac{1}{f}\right)' = -\frac{1}{f^2} \cdot f'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

$$\left(\frac{f}{g}\right)' = \left(f \cdot \frac{1}{g}\right)' \quad \text{poi teorema sul prodotto}$$

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

Esempi di calcolo di derivate

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(e^x)' = e^x$$

$$(\log x)' = \frac{1}{x}$$

$$\textcircled{1} \quad f(x) = \tan x \qquad \tan x = \frac{\sin x}{\cos x}$$

$$(\tan x)' = \frac{\cos x \cdot \cos x - (-\sin x) \cdot \sin x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \begin{array}{l} \nearrow 1 + \frac{\sin^2 x}{\cos^2 x} \\ \searrow \frac{1}{\cos^2 x} \end{array} = 1 + \tan^2 x$$

$$\textcircled{2} \quad f(x) = \sin(x^2)$$

$$f'(x) = \cos(x^2) \cdot 2x$$

$$\textcircled{3} \quad f(x) = \cos(e^x)$$

$$f'(x) = -\sin(e^x) \cdot e^x$$

$$\textcircled{4} \quad f(x) = e^{\cos x}$$

$$f'(x) = e^{\cos x} \cdot (-\sin x)$$

$$\textcircled{5} \quad f(x) = e^x \cdot \cos x \quad f'(x) = e^x \cdot \cos x + e^x (-\sin x) \\ = e^x (\cos x - \sin x)$$

$$\textcircled{6} \quad f(x) = \sin^3(x^2) = 3 \sin^2(x^2) \cdot \cos(x^2) \cdot 2x$$

$\uparrow$ + esterna, $\uparrow$ intermedia $\uparrow$ più
 cioè il cubo cioè il sin interna, cioè x^2

$$\textcircled{7} \quad f(x) = (\sin x)^x \dots$$

$$\textcircled{8} \quad f(x) = \boxed{5^x} = e^{\log(5^x)} = \boxed{e^{x \log 5}} \quad \text{STESSA COSA}$$

$$f'(x) = e^{x \log 5} \cdot \log 5 = 5^x \cdot \log 5$$

$$\textcircled{7} \quad f(x) = (\sin x)^x = e^{x \log(\sin x)}$$

$$f'(x) = e^{x \log(\sin x)} [x \log(\sin x)]'$$

$$= (\sin x)^x \cdot [x \log(\sin x)]'$$

$$= (\sin x)^x \cdot [\log(\sin x) + x (\log(\sin x))']$$

$$= (\sin x)^x \cdot \left[\log(\sin x) + x \frac{1}{\sin x} \cdot \cos x \right]$$

$$\left[(\sin x)^x \right]' = x (\sin x)^{x-1} \cdot \cos x \quad \text{Nooo!!!!}$$

$$\textcircled{9} \quad f(x) = (\sin x)^k \quad f'(x) = k (\sin x)^{k-1} \cdot \cos x$$

$\uparrow$
 SI

EspONENTE FISSO ok, con espONENTE variabile NO.

$$c(x) = f(g(x))$$

$$\underline{\text{Hp}} \quad g(x_0 + h) = g(x_0) + h g'(x_0) + o(h)$$

$$\text{Posto } y_0 = g(x_0), \quad f(y_0 + k) = f(y_0) + k f'(y_0) + o(k)$$

$$c(x_0 + h) = f(g(x_0 + h))$$

$$= f\left(g(x_0) + \underbrace{h g'(x_0) + o(h)}_{k}\right)$$

$$= f(y_0 + k)$$

$$\begin{aligned}
&= f(y_0) + [h g'(x_0) + o(h)] f'(y_0) + o([h]^k) \\
&= f(g(x_0)) + h \boxed{g'(x_0) f'(g(x_0))} + o(h) + o(h)
\end{aligned}$$

Resterebbe da fare che

$$o(h g'(x_0) + o(h)) \Rightarrow o(h)$$