

Come dimostrare che un limite di funzioni non esiste

Ci si basa sul criterio FUNZIONI $\rightarrow$ SUCCESSIONI. Supponiamo

$\lim_{x \rightarrow x_0} f(x) = l$. Sia a_n una qualunque succ. t.-c.
 $a_n \rightarrow x_0$

$$\lim_{n \rightarrow +\infty} f(a_n) = \lim_{x \rightarrow x_0} f(x) = l \quad (\text{stesso } l)$$

$\uparrow$
Pongo $a_n = x$

Quando $n \rightarrow +\infty$,

ho che $x \rightarrow x_0$

Supponiamo che esistano 2 successioni $a_n \rightarrow x_0$
 $b_n \rightarrow x_0$

e supponiamo che

$$f(a_n) \rightarrow l_1 \in \bar{\mathbb{R}} \quad l_1 \neq l_2.$$

$$f(b_n) \rightarrow l_2 \in \bar{\mathbb{R}}$$

Allora $\lim_{x \rightarrow x_0} f(x)$ non esiste (se esistesse e fosse un certo l , dovrei avere $l = l_1 = l_2$).

Esempio $\lim_{x \rightarrow 0^+} \sin \frac{1}{x}$ non esiste.

Dim. Devo trovare 2 succ. $a_n \rightarrow 0^+$ b.c.
 $b_n \rightarrow 0^+$

$$\sin \frac{1}{a_n} \rightarrow l_1$$

$$\sin \frac{1}{b_n} \rightarrow l_2$$

$$l_1 \neq l_2$$

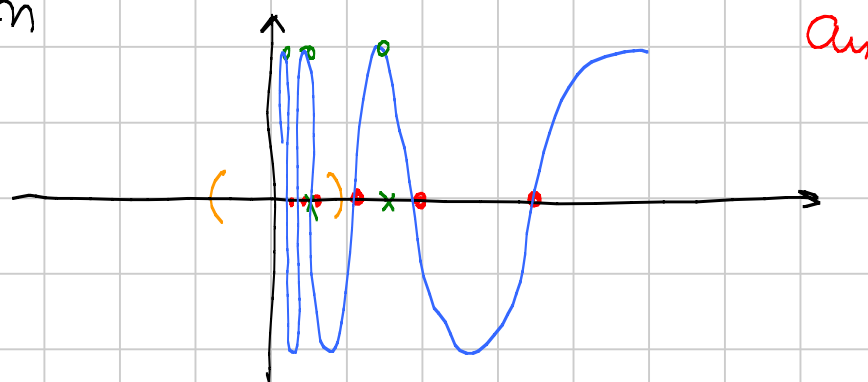
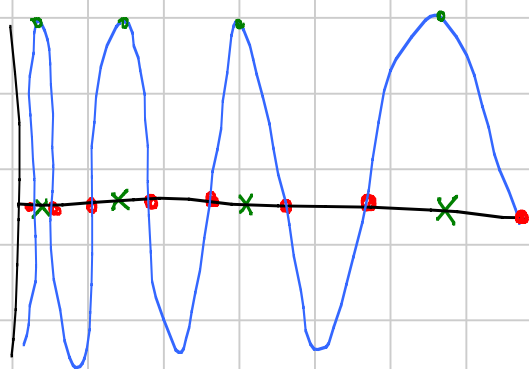
Cerco a_n in modo che sia $\sin \frac{1}{a_n} = 0$, cioè $\frac{1}{a_n} = \pi n$

Quindi $a_n = \frac{1}{\pi n} (\rightarrow 0^+)$.

Cerco b_n in modo che sia

$$\sin \frac{1}{b_n} = 1, \text{ cioè } \frac{1}{b_n} = \frac{\pi}{2} + 2\pi n$$

quindi $b_n = \frac{1}{\frac{\pi}{2} + 2\pi n} \rightarrow 0^+$



Esercizi misti

$$\lim_{x \rightarrow +\infty} \sqrt{x+27} - \sqrt{x+25} \quad [\infty - \infty]$$

$$\left(\sqrt{x+27} - \sqrt{x+25} \right) \frac{\sqrt{x+27} + \sqrt{x+25}}{\sqrt{x+27} + \sqrt{x+25}} = \frac{A^2 - B^2}{\sqrt{x+27} + \sqrt{x+25}} = \frac{\cancel{x+27} - \cancel{x-25}}{\sqrt{x+27} + \sqrt{x+25}}$$

$$= \frac{2}{\sqrt{x+27} + \sqrt{x+25}} \rightarrow 0$$

$$\lim_{x \rightarrow +\infty} \sqrt[3]{x+1} - \sqrt[3]{x} = 0$$

$$(A-B)(A^2+AB+B^2) = A^3-B^3$$

$$\frac{\sqrt[3]{x+1} - \sqrt[3]{x}}{A - B} = \frac{\sqrt[3]{x+1} + \sqrt[3]{x} + \sqrt[3]{x}}{\sqrt[3]{(x+1)^2} + \sqrt[3]{x(x+1)} + \sqrt[3]{x^2}} = \frac{\cancel{x+1} - \cancel{x}}{\sqrt[3]{\dots} + \sqrt[3]{\dots} + \sqrt[3]{\dots}}$$

$\begin{matrix} A^2 & + & AB & + & B^2 \\ \hline & 0 & \end{matrix}$

$\downarrow$
0

$$\lim_{x \rightarrow +\infty} \sqrt{x+5} - \sqrt[3]{x}$$

Raccorlo $\sqrt{x}$

$$\sqrt{x} \left(\frac{\sqrt{x+5}}{\sqrt{x}} - \frac{\sqrt[3]{x}}{\sqrt{x}} \right)$$

$$= \underbrace{\sqrt{x}}_{\rightarrow +\infty} \left(\underbrace{\sqrt{1 + \frac{5}{x}}}_{\rightarrow 0} - \underbrace{\frac{1}{x^{1/6}}}_{\rightarrow 0} \right) \rightarrow +\infty$$

$\downarrow$
1

$$\frac{x^{1/3}}{x^{1/2}} = \frac{1}{x^{\frac{1}{2} - \frac{1}{3}}} = \frac{1}{x^{1/6}}$$

$$\lim_{n \rightarrow +\infty}$$

$$\boxed{\frac{n! \cdot 2^n}{n^n}}$$

a_n

Applico criterio radice

$$\sqrt[n]{a_n} = \sqrt[n]{\frac{n! \cdot 2^n}{n^n}} = \boxed{\frac{\sqrt[n]{n!}}{n}} \cdot 2 \rightarrow \frac{2}{e}$$

$\downarrow \frac{1}{e}$

$$\sqrt[n]{a_n} \rightarrow \frac{2}{e} < 1 \Rightarrow a_n \rightarrow 0$$

$$\lim_{n \rightarrow +\infty}$$

$$\frac{n! \cdot 3^n}{n^n}$$

Radice $\sqrt[n]{a_n} \rightarrow \frac{3}{e} > 1 \Rightarrow a_n \rightarrow +\infty$

— 0 — 0 —

$$\lim_{n \rightarrow +\infty}$$

$$\boxed{\frac{n! \cdot (2n)!}{(3n)!}}$$

a_n

Rapporto

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)! \cdot (2n+2)!}{(3n+3)!}}{\frac{n! \cdot (2n)!}{(3n)!}} =$$

$$= \frac{(n+1)! (2n+2)!}{(3n+3)!} \cdot \frac{(3n)!}{n! (2n)!}$$

$$= \frac{(n+1) \cancel{n!} (2n+2) (2n+1) \cancel{(2n)!}}{(3n+3) (3n+2) (3n+1) \cancel{(3n)!}} \cdot \frac{\cancel{(3n)!}}{\cancel{n!} \cancel{(2n)!}} \xrightarrow{\frac{4n^3}{27n^3}} \frac{4}{27} < 1$$

$$\Rightarrow a_n \rightarrow 0$$

$(2n+2)!$ = prodotto degli interi pos. fino a $2n+2$

$$= (2n+2) (2n+1) \underbrace{(2n) (2n-1) (2n-2) \dots}_{(2n)!}$$

$$\lim_{n \rightarrow +\infty} \sqrt[n]{\frac{n! (2n)!}{(3n)!}} = \frac{4}{27}$$

a_n

RAPPORTO $\rightarrow$ RADICE

Se $\frac{a_{n+1}}{a_n} \rightarrow l$, allora $\sqrt[n]{a_n} \rightarrow l$

Fatto prima $\frac{a_{n+1}}{a_n} \rightarrow \frac{4}{27} \Rightarrow \sqrt[n]{a_n} \rightarrow \frac{4}{27}$

$$\lim_{n \rightarrow +\infty} \sqrt[n]{\frac{5^n + n^5}{3^n + n^7}} = \frac{5}{3}$$

$$\sqrt[n]{\frac{5^n \left(1 + \frac{3^5}{5^3}\right)}{3^n \left(1 + \frac{3^7}{3^3}\right)}} =$$

$$\frac{5}{3} \left(\frac{1 + \frac{3^5}{5^3}}{1 + \frac{3^7}{3^3}} \right)^{\frac{1}{n}}$$

$$\rightarrow \left(\frac{1}{1}\right)^0 = 1$$

$$\lim_{x \rightarrow +\infty} \frac{2^x}{x^8} = +\infty$$

$$\lim_{x \rightarrow +\infty} \frac{x}{\log x} = +\infty$$

$$\lim_{x \rightarrow +\infty} \frac{\log(5^x + x^5)}{x}$$

Brutalweise $\frac{\log(5^x)}{x} =$

$$= \frac{\cancel{x} \log 5}{\cancel{x}} = \log 5$$

Rigorous

$$\frac{\log 5^x \left(1 + \frac{x^5}{5^x}\right)}{x} = \frac{\log 5^x + \log \left(1 + \frac{x^5}{5^x}\right)}{x} = \frac{x \log 5 + \log \left(1 + \frac{x^5}{5^x}\right)}{x}$$

$$= \frac{\cancel{x} \left(\log 5 + \frac{\log \left(1 + \frac{x^5}{5^x}\right)}{x} \right)}{\cancel{x}} \rightarrow \frac{\log 1}{\infty} = \frac{0}{\infty} = 0 \rightarrow \log 5$$

$$\lim_{x \rightarrow 0} \frac{e^{\sqrt{x}}}{x^{200}}$$

$$x^{200} = e^{\boxed{\dots}} = \overbrace{\log x}^{(200)} = 200 \log x$$

$$\frac{e^{\sqrt{x}}}{x^{200}} = \frac{e^{\sqrt{x}}}{e^{200 \log x}} = e^{\sqrt{x} - 200 \log x} \rightarrow e^{+\infty} = +\infty$$

Basta fare il limite dell'esponente

$$\sqrt{x} - 200 \log x = \underbrace{\sqrt{x}}_{+\infty} \left(1 - 200 \underbrace{\frac{\log x}{\sqrt{x}}}_{0} \right) = +\infty - 1 = +\infty$$