

MAT 1 TLC

ORA 18

$$\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^x = e \quad \text{Quindi}$$

[è come se avessi fatto
 $y = \left(1 + \frac{1}{x}\right)^x$]

$$\lim_{x \rightarrow +\infty} \log \left[\left(1 + \frac{1}{x}\right)^x \right] = 1 = \log e$$

$$\lim_{x \rightarrow +\infty} x \log \left(1 + \frac{1}{x}\right) = 1$$

Pongo $\frac{1}{x} = y$. Quando $x \rightarrow +\infty$,
ho che $y \rightarrow 0^+$. Inoltre $x = \frac{1}{y}$

$$= \lim_{y \rightarrow 0^+} \frac{1}{y} \log(1+y) = 1$$

$$\lim_{y \rightarrow 0^-} \frac{1}{y} \log(1+y) = \lim_{x \rightarrow -\infty}$$

$$x \log \left(1 + \frac{1}{x}\right) =$$

$$= \lim_{x \rightarrow -\infty} \log \left[\left(1 + \frac{1}{x} \right)^x \right] = \log e = 1$$

↓
e anche per
 $x \rightarrow -\infty$

$$1 = \lim_{x \rightarrow 0} \frac{\log(1+x)}{x}$$

Pongo $\log(1+x) = y$.

Quando $x \rightarrow 0$, ho che $y \rightarrow \log 1 = 0$.

Inoltre $1+x = e^y$, quindi $x = e^y - 1$

$$= \lim_{y \rightarrow 0} \frac{y}{e^y - 1}$$

— 0 — 0 —

$$\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^x = e$$

Perché? Non ve lo dico, ma si deduce con passaggi opportuni, in dall' analogo limite con le succ.

— o — o —

$$\lim_{x \rightarrow 0^+} \frac{\sin x}{x} = 1. \text{ Supponiamo di sapere che}$$

$$\sin x \leq x \leq \tan x$$

$$\text{per ogni } x \in \left(0, \frac{\pi}{2}\right)$$

Divido per $\sin x$
cancellando i
versi

$$1 \leq \frac{x}{\sin x} \leq \frac{1}{\cos x}$$

↓ ↓ ↓
1 1 1

Quando $x \rightarrow 0^+$,
entra in questa zona.

$$\frac{\tan x}{\sin x} = \frac{\cancel{\sin x}}{\cos x} \cdot \frac{1}{\cancel{\sin x}}$$

Per $x \rightarrow 0^-$. La funzione $f(x) = \frac{\sin x}{x}$ è pari.

$$f(-x) = \frac{\sin(-x)}{-x} = \frac{-\sin x}{-x} = \frac{\sin x}{x} = f(x)$$

sin è
DISPARI

Se $f(x)$ è pari, allora $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x)$

$$\lim_{x \rightarrow 0^-} f(x) =$$

Pongo $y = -x$. Quando $x \rightarrow 0^-$,
ho che $y \rightarrow 0^+$

$$= \lim_{y \rightarrow 0^+} f(-y) = \lim_{y \rightarrow 0^+} f(y)$$

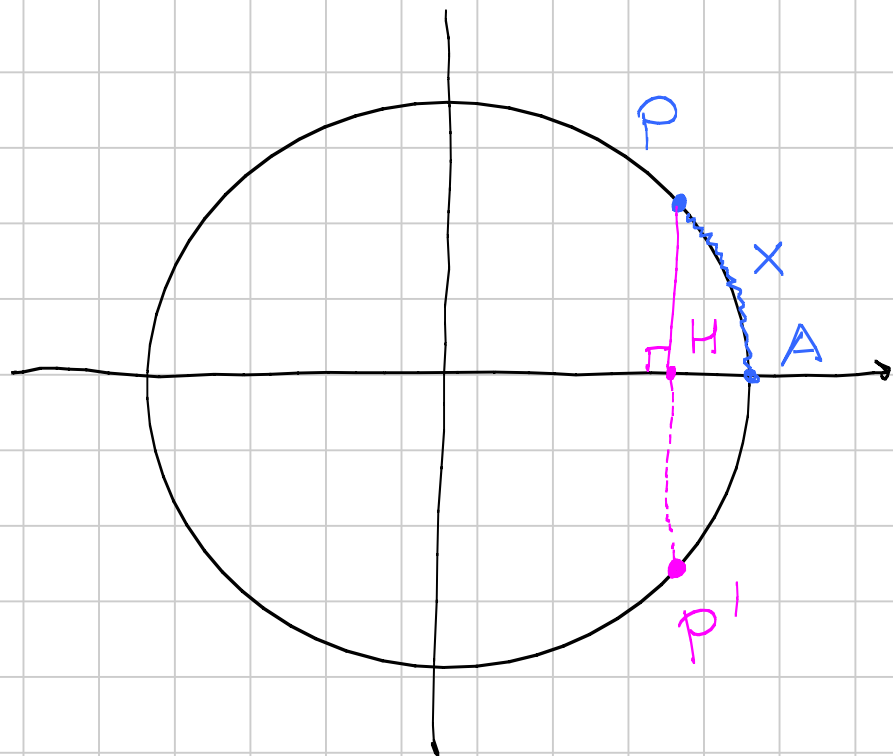
f è PARI

Es. f dispari $\Rightarrow$

$$\lim_{x \rightarrow 0^+} f(x) = - \lim_{x \rightarrow 0^-} f(x)$$

Resta da dimostrare che

$$\sin x \leq x \leq \tan x \quad \forall x \in (0, \frac{\pi}{2})$$



$x = \text{lungh. arco AP}$

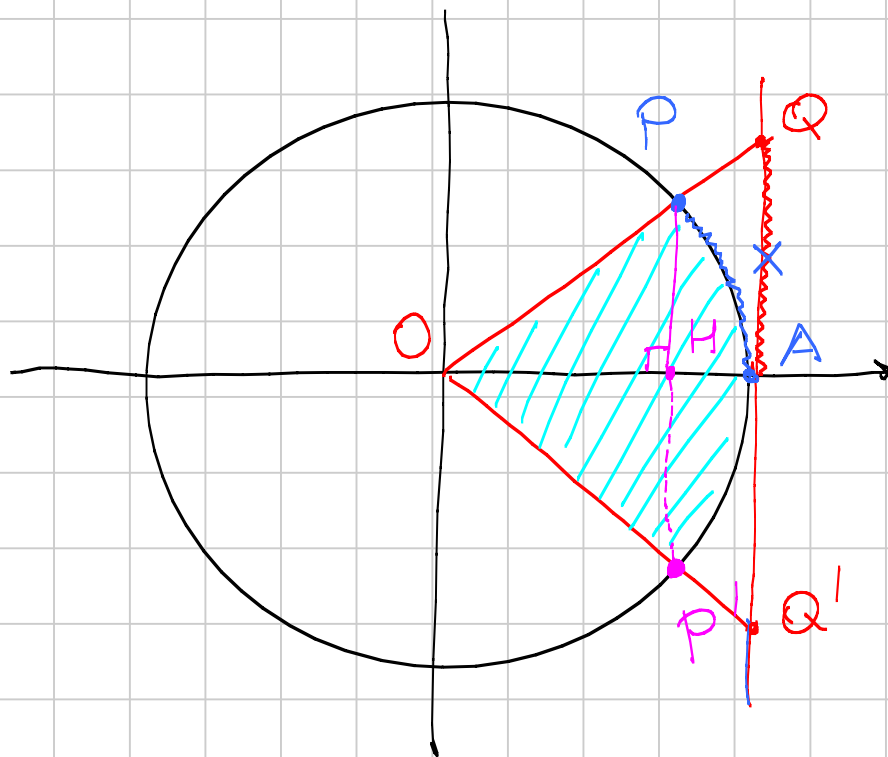
$\sin x = \text{lungh. di PH}$

$2x = \text{lungh. arco } \widehat{PP'}$

$2\sin x = \text{lungh. segmento } \overline{PP'}$

segm. $\leq$ arco

$$\cancel{2}\sin x \leq \cancel{2}x$$



$$\begin{aligned}
 \text{Area } OQQ' &= \frac{1}{2} QQ' \cdot OA \\
 &= \frac{1}{2} 2 \tan x \cdot 1 \\
 &= \tan x
 \end{aligned}$$

$$\text{Area Setto} = \frac{\pi \cdot 2x}{2\pi} = x$$

$$\begin{aligned}
 \text{Area Setto} &\leq \text{Area } OQQ' \\
 x &\leq \tan x
 \end{aligned}$$

$$\begin{aligned}
 \text{Area Settore} : \text{Area Cerchio} &= \text{angolo settore} : 2\pi \\
 \text{Area Setto} : \pi &= 2x : 2\pi
 \end{aligned}$$

Esercizio 1

$$\lim_{x \rightarrow 0} \frac{e^{\tan x} - 1}{x} = \lim_{x \rightarrow 0} \boxed{\frac{e^{\tan x} - 1}{\tan x}} \cdot \boxed{\frac{\tan x}{x}} = 1$$

$$\downarrow \begin{matrix} y = \tan x \\ y \rightarrow 0 \end{matrix}$$

$$\downarrow 1$$

$$\lim_{y \rightarrow 0} \frac{e^y - 1}{y} = 1$$

Es. 2

$$\lim_{x \rightarrow 0} \frac{\cos x - e^x}{\sin(\arctan x)} = -1$$

$$\frac{\cos x - e^x}{\sin(\arctan x)} \cdot \frac{\arctan x}{\arctan x} \cdot \frac{x}{x}$$

$$\frac{\arctan x}{\sin(\arctan x)}$$

$$\downarrow y = \arctan x$$

$$\downarrow 1$$

$$\frac{x}{\arctan x}$$

$$\downarrow 1$$

$$\frac{\cos x - e^x}{x}$$

||

$$\rightarrow -1 \rightarrow -1$$

$$\frac{\cos x - 1 + 1 - e^x}{x}$$

||

$$\frac{\cos x - 1}{x} + \frac{1 - e^x}{x}$$

$$\boxed{x} \cdot \frac{\cos x - 1}{x^2} + \frac{1 - e^x}{x}$$

$$\downarrow 0 \quad \cdot \quad -\frac{1}{2} \quad + \quad -1$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{x - \frac{\pi}{2}}$$

$$\left[\frac{0}{0} \right]$$

Pongo $x - \frac{\pi}{2} = y$

Quando $x \rightarrow \frac{\pi}{2}$, ho che $y \rightarrow 0$

$$\lim_{y \rightarrow 0} \frac{\cos\left(\frac{\pi}{2} + y\right)}{y}$$

$\equiv$ [Precorso:
archi
associati]

$$x = \frac{\pi}{2} + y$$

$$\equiv \lim_{y \rightarrow 0} \frac{-\sin y}{y} = -1,$$