

MAT 1 TLC

ORA 5

Consideriamo funzioni $f: \mathbb{R} \rightarrow \mathbb{R}$

Def. f si dice **PARI** se $f(x) = f(-x) \quad \forall x \in \mathbb{R}$

f si dice **DISPARI** se $f(-x) = -f(x) \quad \forall x \in \mathbb{R}$

f si dice **PERIODICA** se $\exists T > 0$ tale che

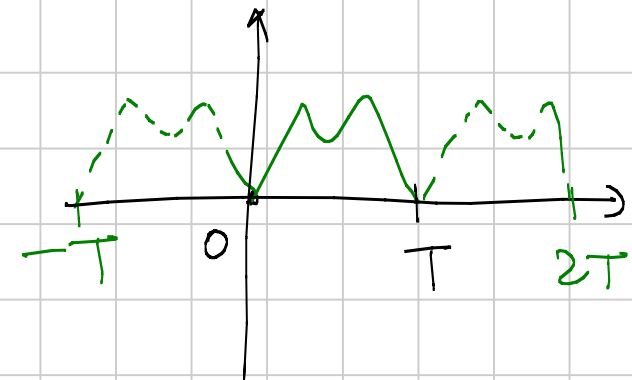
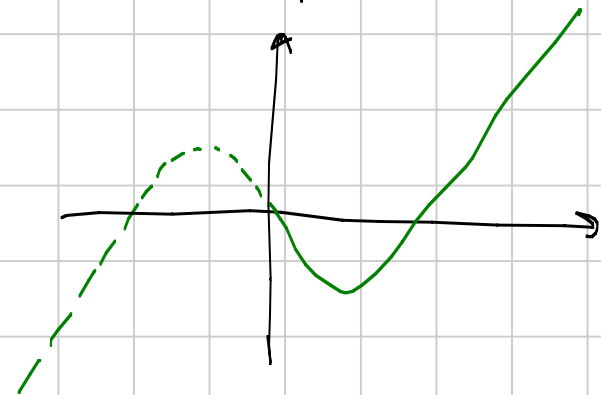
$$f(x+T) = f(x) \quad \forall x \in \mathbb{R}$$

In tal caso il più piccolo $T > 0$ con questa proprietà (se esiste) si dice **MINIMO PERIODO**

Graficamente

f pari $\Leftrightarrow$ grafico simmetrico
risp. asse y

f dispari $\Leftrightarrow$ grafico simm.
rispetto all'origine



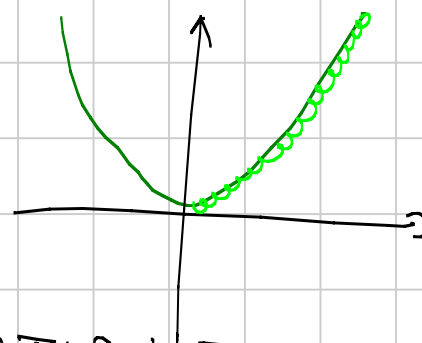
FUNZIONI ELEMENTARI

$$f(x) = x^2$$

PARI

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

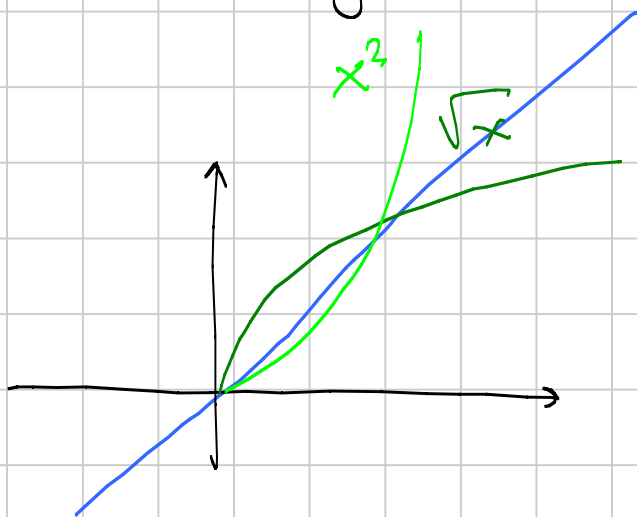
NO INJ.
NO SURG.



$$f: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0} \quad \text{INVERTIBILE.}$$

La sua inversa è una funzione $g: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$

$$g(x) = \sqrt{x}$$

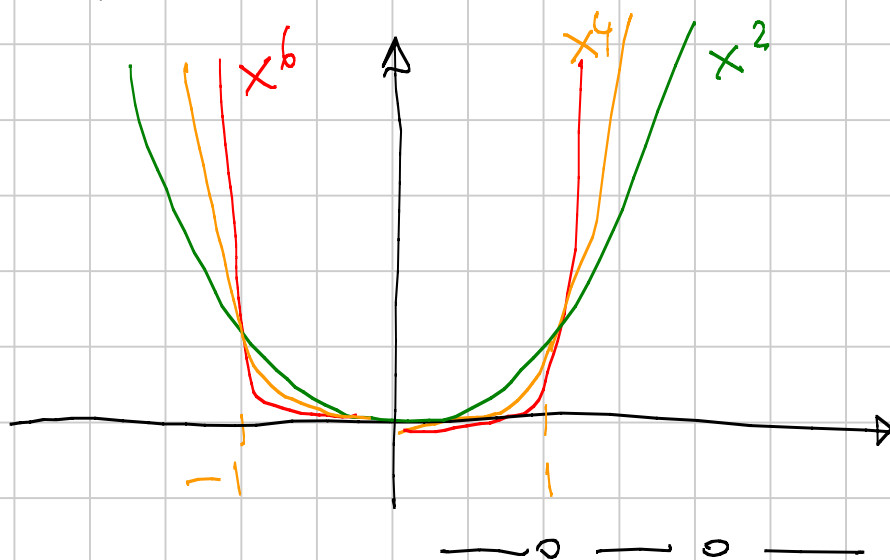


N.B. Per definizione $\sqrt{x}$ accetta
in INPUT solo valori $x \geq 0$ e restituisce
in OUTPUT valori ≥ 0 .

N.B. I grafici di una funzione e
della sua inversa sono simm.
rispetto bisettrice $y = x$

$$f(x) = x^4, x^6, x^8, \dots$$

stessa cosa $\leadsto \sqrt[4]{x}, \sqrt[6]{x}, \sqrt[8]{x}$



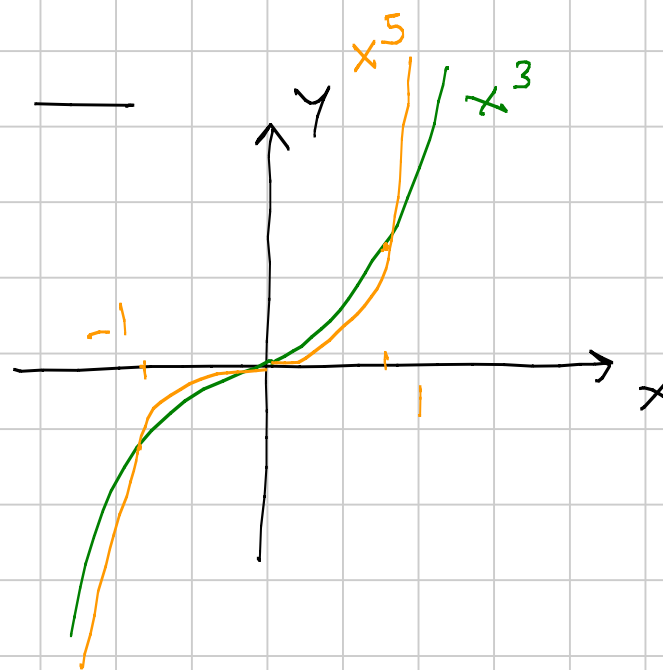
$$f(x) = x^3$$

← FUNZIONE
DISPARI

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

INJ e SURG

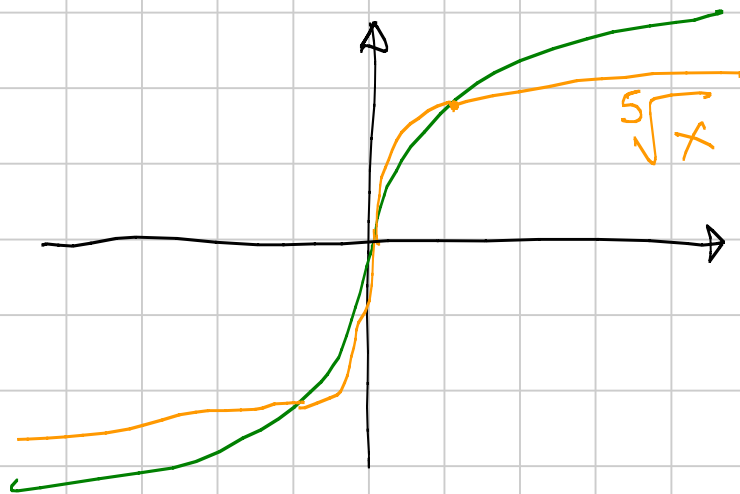
$\Downarrow$
INVERTIBILE



L'inversa è una funzione

$$g: \mathbb{R} \rightarrow \mathbb{R}$$

$$g(x) = \sqrt[3]{x}$$



$\sqrt[3]{x}$ accetta in INPUT
sia pos. sia neg. e
restituisce in OUTPUT
valori con lo stesso segno

Stessa cosa per $x^5, x^7, x^9, \dots$

$$\sqrt[5]{x}, \sqrt[7]{x}, \sqrt[9]{x}, \dots$$

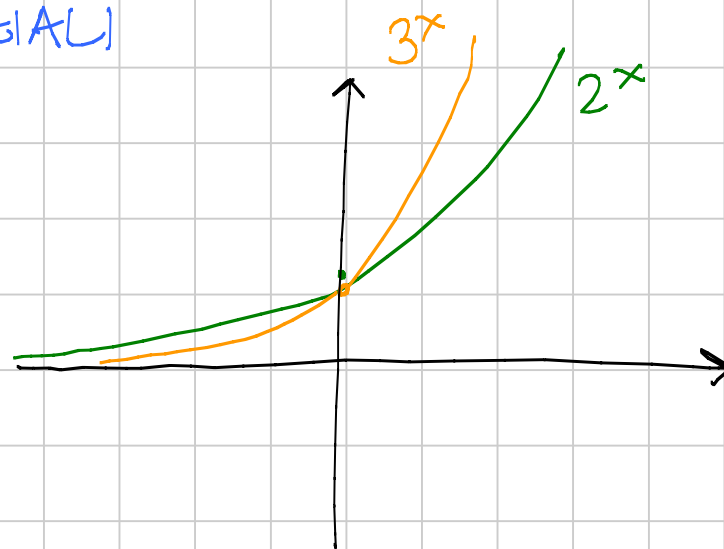
FUNZIONI ESPONENZIALI

$$f(x) = 2^x$$

N.B. x base, espou = numero $\leftarrow$ POTENZE

x esponente $\leftarrow$ ESPONENZIALI

$f: \mathbb{R} \rightarrow \mathbb{R}$ SI INS.
NO SURG.



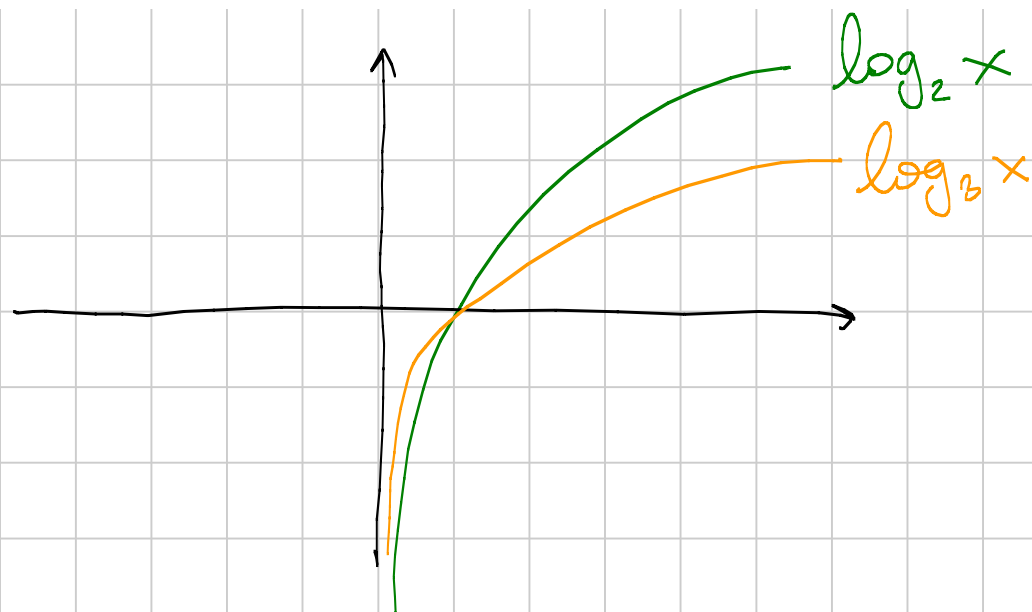
$f: \mathbb{R} \rightarrow \mathbb{R}_{>0}$

INVERTIBILE

La sua inversa è una funzione $g: \mathbb{R}_{>0} \rightarrow \mathbb{R}$

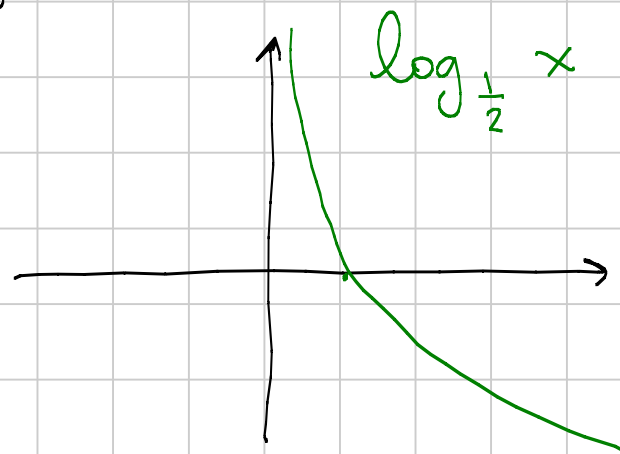
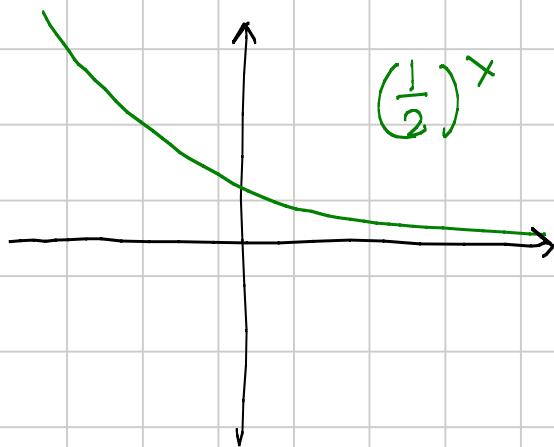
$$g(x) = \log_2 x$$

$\uparrow$
 $\log_2 x$ accetta in
INPUT valori > 0



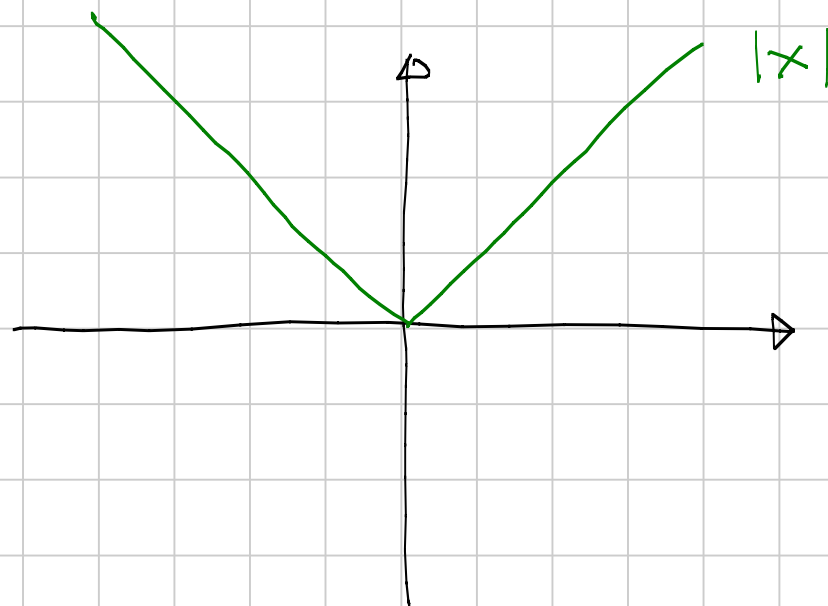
Stessa per
funzioni
 a^x con $a > 1$

a^x con esponente $0 < a < 1$



$$f(x) = |x|$$

$$|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$



Funzione pari

$$f: \mathbb{R} \rightarrow \mathbb{R} \quad \begin{array}{l} \text{NO INT.} \\ \text{NO SURG.} \end{array}$$

$$f: \mathbb{R} \rightarrow \mathbb{R}_{\geq 0} \quad \begin{array}{l} \text{NO INT.} \\ \text{SI SURG.} \end{array}$$

FUNZIONI TRIGONOMETRICHE

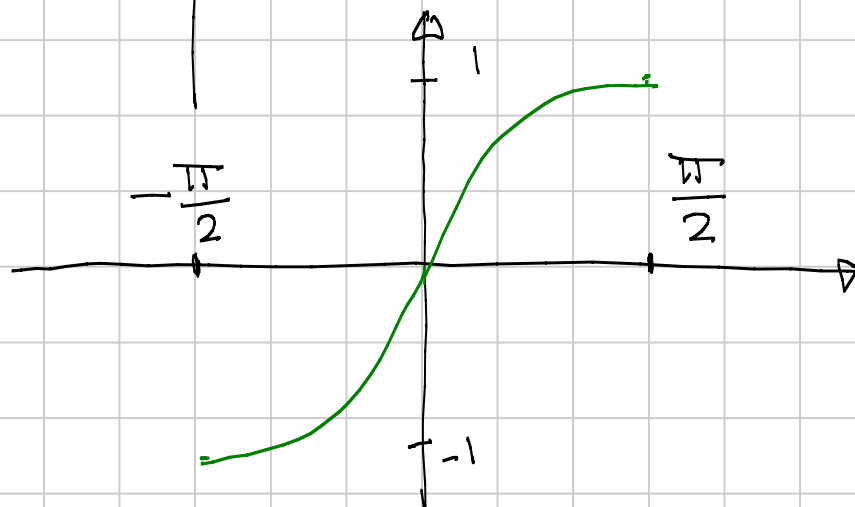
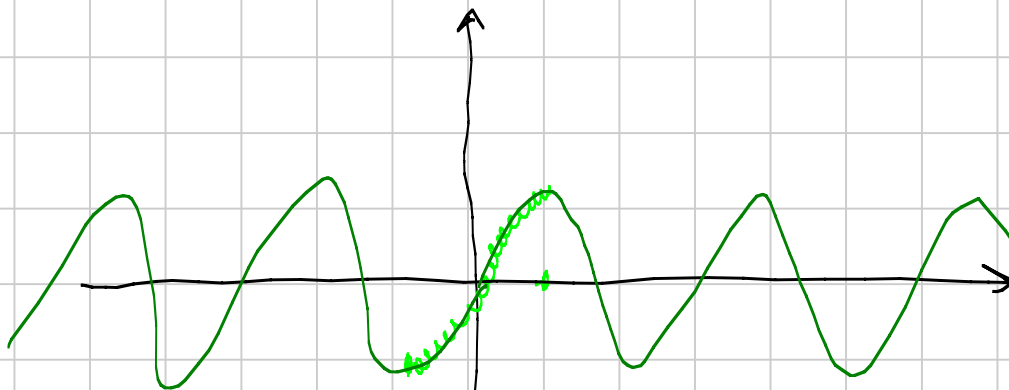
$$f(x) = \sin x$$

DISPARI

$f: \mathbb{R} \rightarrow \mathbb{R}$ NO INJ.
NO SURG,

$$f: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightarrow [-1, 1]$$

INVERTIBILE

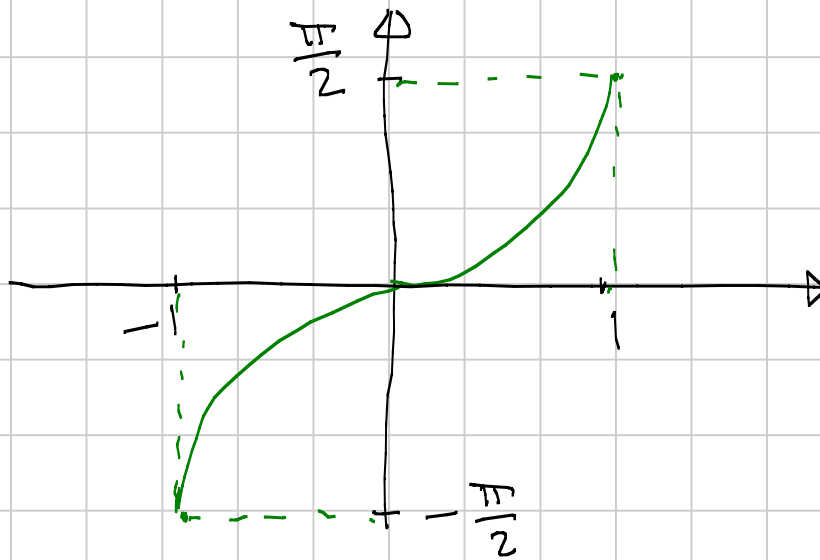


La sua inversa è $g: [-1, 1] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$g(x) = \arcsin(x)$$

INPUT: valori $x \in [-1, 1]$

OUTPUT: valori
in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

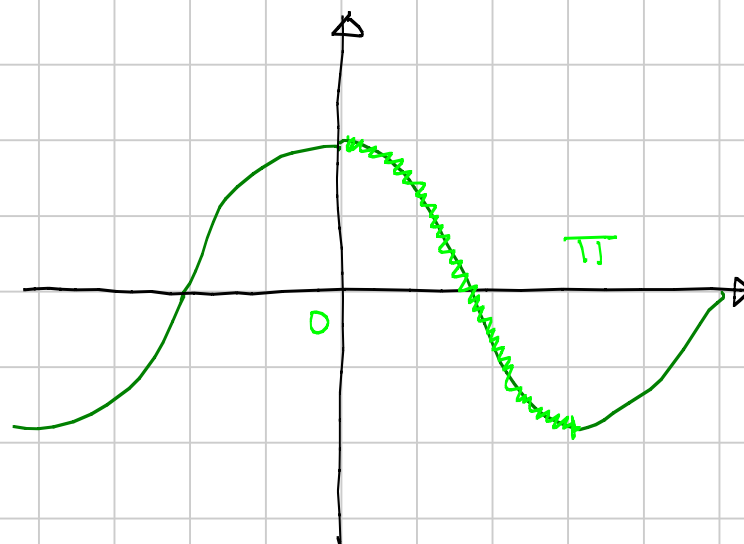


$$f(x) = \cos x$$

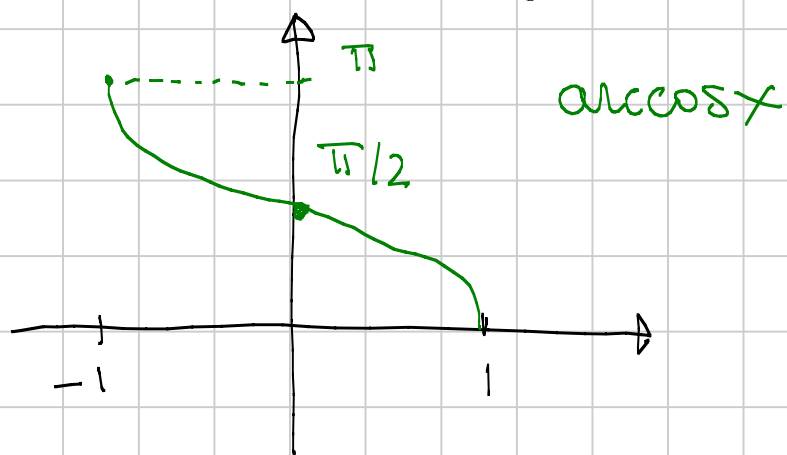
PAR)

$$f: [0, \pi] \rightarrow [-1, 1]$$

INVERTIBILE



è l'inversa $g: [-1, 1] \rightarrow [0, \pi]$ e' $g(x) = \arccos x$

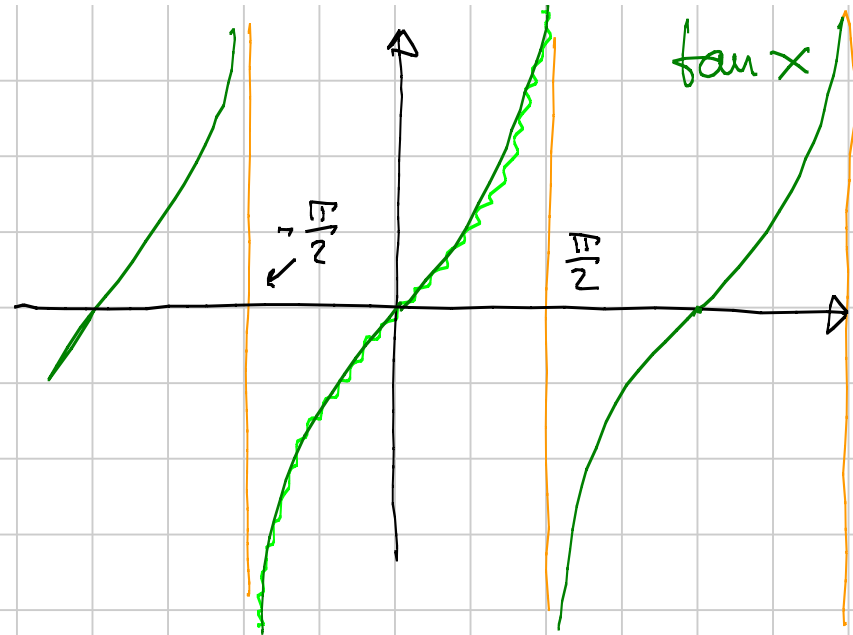


$$f(x) = \tan x$$

DISPARI

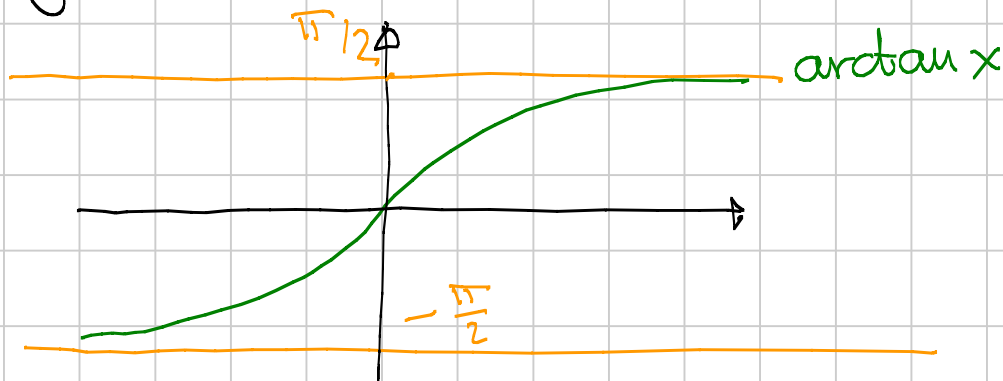
$$f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \mathbb{R}$$

$$\begin{matrix} \updownarrow \\ \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \end{matrix}$$



INVERTIBILE. L'inversa è una
funzione $g: \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$g(x) = \arctan x$$



INPUT: ogni $x \in \mathbb{R}$
OUTPUT: solo valori
in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Esempi

$$\sin(\arcsin(\frac{1}{2})) = \frac{1}{2}$$

$$\arcsin(\sin 1) = 1$$

$$\arccos(\cos 1) = 1$$

$$\arcsin(\sin 2) = 2$$

$$\arccos(\cos 2) = 2$$

$$\arccos(\cos 3) = 3$$

$$\arccos(\cos 4) = 4$$

SI

←

←

←

←

←

NO

SI

SI

NO

arcsin NON
SPOTA ROBA
 $> \pi/2$

arccos SPOTA
ROBA $\leq \pi$
Non può
DARE 4.