

$$x^2 + y^2 - xy$$

$$\begin{pmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{pmatrix}$$

$$D = \frac{5}{4}$$

$$S = 2$$

++

Def. pos.

$$x^2 + y^2 - 2xy$$

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\lambda = 0$$

$$\lambda = 2$$

+ 0

Semidef. +

$$x^2 + y^2 - 3xy$$

$$\begin{pmatrix} 1 & -\frac{3}{2} \\ -\frac{3}{2} & 1 \end{pmatrix}$$

$$D = -\frac{5}{4}$$

+ -

Indef.

$$x^2 + 2y^2 - 3xy$$

$$\begin{pmatrix} 1 & -\frac{3}{2} \\ -\frac{3}{2} & 2 \end{pmatrix}$$

$$D = -\frac{1}{4}$$

+ -

Indef.

$$x^2 + y^2 - xy = x^2 - xy + y^2 = x^2 - xy + \frac{1}{4}y^2 - \frac{1}{4}y^2 + y^2$$

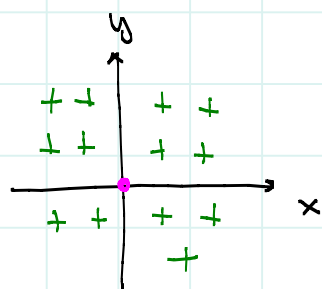
$$= \left(x - \frac{y}{2}\right)^2 + \frac{3}{4}y^2 = \left(x - \frac{y}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}y\right)^2$$

Dove si annulla?

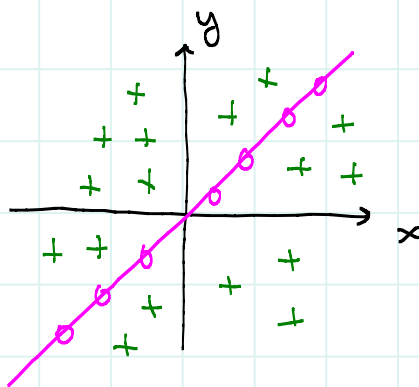
Dove sono entrambi = 0

Ma essendo lin. indep.

questo accade $\Leftrightarrow x = y = 0$



$$x^2 + y^2 - 2xy = (x - y)^2$$

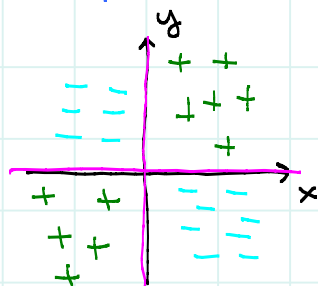


$$7xy$$

$$\begin{pmatrix} 0 & \frac{7}{2} \\ \frac{7}{2} & 0 \end{pmatrix}$$

$$D = \text{neg.}$$

+ -



$$-x^2 - 2y^2 + 10xy$$

$$\begin{pmatrix} -1 & 5 \\ 5 & -2 \end{pmatrix}$$

$$D = -23$$

+ -

$$-y^2 - 7xy$$

$$\begin{pmatrix} 0 & -\frac{7}{2} \\ -\frac{7}{2} & -1 \end{pmatrix}$$

$$D = \text{neg.}$$

+ -

$$3x^2 + 16y^2 - 14xy$$

$$\begin{pmatrix} 3 & -7 \\ -7 & 16 \end{pmatrix}$$

$$D = 48 - 49$$

+ -

$$(x + ay)^2 - (x - ay)^2 \leadsto \text{ho a}$$

$$7xy = (x + \frac{7}{4}y)^2 - (x - \frac{7}{4}y)^2$$

$$x^2 + \frac{7}{2}xy + \frac{49}{16}y^2 - x^2 + \frac{7}{2}xy - \frac{49}{16}y^2$$

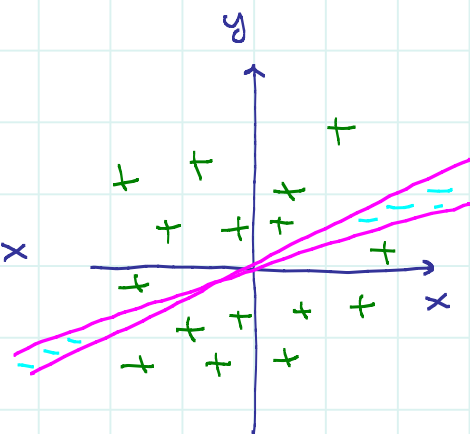
Nel 4° esempio, trovare un vettore x, y t.c. $q(x, y) < 0$

$$\begin{aligned} 3x^2 + 16y^2 - 14xy &= 16y^2 - 14xy + 3x^2 \\ &= 16y^2 - 14xy + \frac{49}{16}x^2 - \frac{49}{16}x^2 + 3x^2 \\ &= \left(4y - \frac{7}{4}x\right)^2 - \frac{1}{16}x^2 \end{aligned}$$

Se voglio che venga negativo prendo $x=16$ e $y=7$ in modo che si annulli il 1° termine.

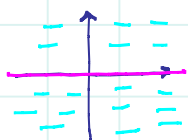
Facciamo il disegno dei segni. Si annulla dove

$$4y - \frac{7}{4}x = \pm \frac{1}{4}x \quad \leadsto \quad \begin{aligned} 4y - 2x &= 0 & y &= \frac{x}{2} \\ 4y - \frac{3}{2}x &= 0 & y &= \frac{3}{8}x \end{aligned}$$



$$-y^2$$

$$-0$$



semidef. neg.

$$x^2 + xy$$

$$+ -$$

$$5x^2 + 7y^2 + 12xy$$

$$+ -$$

$$-3x^2 - 16y^2 - 14xy$$

praticamente uguale ad uno dei precedenti

$$x^2 + xy = x^2 + xy + \frac{y^2}{4} - \frac{y^2}{4} = \left(x + \frac{y}{2}\right)^2 - \left(\frac{y}{2}\right)^2$$

$$\begin{aligned} 5x^2 + 7y^2 + 12xy &= 5x^2 + 12xy + 7y^2 \\ &= 5x^2 + 2\sqrt{5}x \cdot \frac{6}{\sqrt{5}}y + \frac{36}{5}y^2 - \frac{36}{5}y^2 + 7y^2 \\ &= \left(\sqrt{5}x + \frac{6}{\sqrt{5}}y\right)^2 - \frac{1}{5}y^2 \end{aligned}$$

$$x^2 + y^2 + yz$$

$$x^2 + y^2 - 2yz$$

$$x^2 + y^2 + 3yz$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{pmatrix}$$

$$\text{Det} = -\frac{1}{4}$$

++-

~~---~~ $\text{Tr} > 0$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\text{Det} = -1$$

++-

~~---~~ $\text{Tr} > 0$

uguale al
precedente

$$x^2 + y^2 + yz = x^2 + y^2 + yz + \frac{z^2}{4} - \frac{z^2}{4} = \oplus x^2 \oplus \left(y + \frac{z}{2}\right)^2 \ominus \frac{z^2}{4}$$

$$x^2 + y^2 - 2yz = x^2 + y^2 - 2yz + z^2 - z^2 = x^2 + (y - z)^2 - z^2$$

Achtung! $q(x, y, z) = 2x^2 - y^2 - 2yz - z^2$

$$q(x, y, z) = x^2 + x^2 - (y + z)^2 \quad ++-$$

NO! NO! NO! I primi 2 quadrati NON sono lin. indep!!!

$$q(x, y, z) = 2x^2 - (y + z)^2 = (\sqrt{2}x) - (y + z)^2 \quad + - 0$$

Stesso con la matrice

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 2-\lambda & 0 & 0 \\ 0 & -1-\lambda & -1 \\ 0 & -1 & -1-\lambda \end{pmatrix}$$

$$p_\lambda(\lambda) = (2-\lambda)[\lambda^2 + 2\lambda - 1] = \lambda(\lambda+2)(2-\lambda)$$

↪ autovalori: $\lambda = 0, \lambda = 2, \lambda = -2$

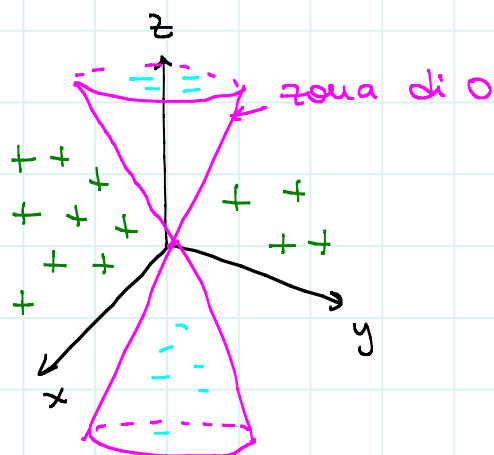
Semplice esempio con disegno

$$q(x, y, z) = x^2 + y^2 - z^2 \quad ++-$$

Dove si annulla? Dove $z^2 = x^2 + y^2$

$$z = \pm \sqrt{x^2 + y^2}$$

distanza dall'origine in $\mathbb{R}^2$



Segui - : dentro al cono

Segui + : fuori dal cono

Ci sono dei piani (s.sp. di dim. 2) contenuti nella zona +

Ci sono solo rette (s.sp. di dim. 1) contenuti nella zona -.

$$x^2 + 2y^2 + 3z^2 + 4xy + 5xz + 6yz$$

$$x^2 - 2y^2 + 3z^2 + 4xy + 5xz + 6yz$$

$$\underline{x^2 + 4xy + 5xz + 2y^2 + 3z^2 + 6yz}$$

quelli con la x

$$\underline{\left(x + 2y + \frac{5}{2}z\right)^2 - 4y^2 - \frac{25}{4}z^2 - 10yz} + 2y^2 + 3z^2 + 6yz$$

uguale ai termini con la x
all'inizio

quello che resta ha solo y e z $\leadsto$ come fosse in dim 2

— 0 — 0 —