

$$\begin{pmatrix} 0 & -1 \\ 4 & 0 \end{pmatrix}$$

$$S=0$$

$$P=4$$

$$\lambda = \pm 2i$$

$$\begin{pmatrix} 1 & -3 \\ -1 & 3 \end{pmatrix}$$

$$S=4$$

$$P=0$$

$$\lambda = \begin{matrix} 0 \\ 4 \end{matrix}$$

$$\begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$$

$$S=4$$

$$P=4$$

$$\lambda = 2, 2$$

$$m_a = 2$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 3 \end{pmatrix}$$

$$S=4$$

$$P=3$$

$$\lambda = \begin{matrix} 1 \\ 3 \end{matrix}$$

Matrice triang. sup.
autoval = tizi sulla
diagonale

$$\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

$$S=0$$

$$P=0$$

$$\lambda = 0, 0$$

$$m_a = 2 \quad m_g = 1$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix}$$

Forma canonica
su $\mathbb{C}$

$$\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$$

Forma canonica
su $\mathbb{R}$

$$\begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

Facciamo con una la 3ª matrice

$$\begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$$

$\lambda = 2$ con $m_a = 2$. Calcolo $m_g(2)$

$$A - 2Id = \begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix} \rightsquigarrow \text{rank} = 1 \rightsquigarrow m_g = \dim(\ker) = 1$$

$\Rightarrow$ NO diag. e JORDAN con un unico blocco

$$J = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$$

Come trovo la M di passaggio? Deve succedere

$$Av_1 = 2v_1$$

$$Av_2 = 2v_2 + v_1$$

$$v_1 \in \ker(A - 2Id) = \text{span}((1, 1))$$

$$v_1 = (1, 1)$$

Per v_2 faccio il bovino

$$v_2 = (x, y)$$

$$\begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$x + y = 2x + 1$$

$$-x + y = 1$$

$$-x + 3y = 2y + 1$$

$$-x + y = 1$$

Posso prendere $v_2 = (0, 1)$. Quindi $M = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

Verifica: $M^{-1} A M = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 \\ -1 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & -2 \\ 5 & -1 \end{pmatrix} \quad \begin{pmatrix} 3 & 0 \\ 3 & 3 \end{pmatrix} \quad \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S = 0$$

$$P = -9$$

$$S = 0$$

$$P = 9$$

$$\lambda = \pm 3i$$

$$\lambda = 3, 3$$

$$w_a = 2$$

$$w_g = 1$$

$$S = 6$$

$$P = 0$$

$$\lambda = 0, 6$$

$$\begin{pmatrix} 3 & 0 \\ 0 & -3 \end{pmatrix}$$

$$\begin{pmatrix} 3i & 0 \\ 0 & -3i \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 6 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

va in 6
volte se stesso

va in
0 (volte
se stesso)

— 0 —

$$\begin{pmatrix} -1 & 2 \\ 8 & -1 \end{pmatrix}$$

$$S = -2$$

$$P = -15$$

$$\lambda = 3, -5$$

$$\begin{pmatrix} -1 & 2 \\ -8 & -1 \end{pmatrix}$$

$$S = -2$$

$$P = 17$$

$$\lambda = -1 \pm 4i$$

$$\begin{pmatrix} 0 & 3 \\ 1 & 0 \end{pmatrix}$$

$$S = 0$$

$$P = -3$$

$$\lambda = \pm \sqrt{3}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$S = 5$$

$$P = -2$$

$$\lambda = \frac{5 \pm \sqrt{33}}{2}$$

$$\begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix}$$

$$S = 5$$

$$P = 10$$

$$\lambda = \frac{5 \pm i\sqrt{15}}{2}$$

$$\begin{pmatrix} 3 & 0 \\ 0 & -5 \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{3} & 0 \\ 0 & -\sqrt{3} \end{pmatrix}$$

Diagon.

$$\begin{pmatrix} \frac{5}{2} & \frac{\sqrt{15}}{2} \\ -\frac{\sqrt{15}}{2} & \frac{5}{2} \end{pmatrix}$$

$$S = -2 \quad P = 17 \quad \lambda^2 + 2\lambda + 17 = 0$$

$$\lambda = -1 \pm \sqrt{1-17} = -1 \pm \sqrt{-16} = -1 \pm 4i$$

$$S = 5 \quad P = -2 \quad \lambda^2 - 5\lambda - 2 = 0$$

$$\lambda = \frac{5 \pm \sqrt{25+8}}{2} = \frac{5 \pm \sqrt{33}}{2}$$

$$S=5 \quad P=10 \quad \lambda^2 - 5\lambda + 10 = 0 \quad \lambda = \frac{5 \pm \sqrt{25-40}}{2} = \frac{5 \pm i\sqrt{15}}{2}$$

Facciamo le M di passaggio nel caso

$$\begin{pmatrix} -1 & 2 \\ -8 & -1 \end{pmatrix} \quad \text{autovalori} \quad \lambda = -1 \pm 4i$$

JORDAN complessa

JORDAN complessa $\begin{pmatrix} -1+4i & 0 \\ 0 & -1-4i \end{pmatrix} = J_c$

$$\boxed{\lambda = -1 + 4i} \quad A - \lambda \text{Id} = \begin{pmatrix} 4i & 2 \\ -8 & 4i \end{pmatrix} \quad \ker = \text{Span}((1, -2i))$$

$$\boxed{\lambda = -1 - 4i} \quad \dots \quad \ker = \text{Span}((1, 2i))$$

M di passaggio complessa:

$$M = \begin{pmatrix} 1 & 1 \\ -2i & 2i \end{pmatrix}$$

Verifica: $M^{-1}AM = J_C$

JORDAN male

$$\begin{pmatrix} -1 & 4 \\ -4 & -1 \end{pmatrix} = J_R$$

↑ [Seguo conetto dopo video]

Vecchio v_1 complesso = $(1, 0) + i(0, -2)$

$\uparrow$ $\uparrow$

movi v₁, v₂

$$M = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix} \quad M^{-1} = \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$$

Verifica: $M^{-1} A M = \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} -1 & 2 \\ -8 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} -1 & -4 \\ -8 & 2 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -4 \\ 4 & -1 \end{pmatrix}$$

$$b=4 \quad b=-4$$

↑

va bene anche questa $\lambda = -1 \pm 4i$ ☺

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -5 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{Forma canonica della 1ª} \quad M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

Rango = 1 $\Rightarrow$ Dim(ker) = 2 $\Rightarrow$ $m_g(0) = 2 \Rightarrow \lambda = 0$ è autovalore almeno doppio

Tr = 3 $\Rightarrow$ gli autovalori sono 0, 0, 3

$m_g = 2$
 $m_a = 2 \Rightarrow$ diagonalizzabile

In questo caso, essendo la matrice simmetrica, sappiamo che possiamo anche trovare M ortogonale! Il che equivale a trovare U_1, U_2, U_3 ORTONORMALI t.c. $AU_1 = 3U_1$ $AU_2 = 0$ $AU_3 = 0$.

Come possiamo fare?

Intanto cerchiamo ORTOGONALI

$$U_1 = (1, 1, 1)$$

$$U_2 = (1, -1, 0) \leftarrow \text{sta nel ker ed è } \perp \text{ a } U_1$$

$$U_3 = (1, 1, -2) \leftarrow \text{Formula misteriosa}$$

* * *

1 1 1

1 -1 0

Dividiamo per le norme trovo

$$M = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \end{pmatrix}$$

↓↓↓

$$\text{Verifica} = M^T A M = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

— 0 — 0 —