Esame di ALGEBRA LINEARE - anno accademico 2019/2020
Corso di laurea in Ingegneria Gestionale
Prova scritta del 28/1/2020
TEMPO A DISPOSIZIONE: 120 minuti

	MARCO	
(Cognome)	(Nome)	(Numero di matricola)

PRIMA PARTE

PUNTEGGIO : risposta mancante = 0 ; risposta esatta = +1 risposta sbagliata = -1

calcoli e spiegazioni non sono richiesti

- $z = 3 + i4$, $w = 1 + i2 \Rightarrow Re(z \cdot w) =$ $\boxed{-5}$ • $z = 1 + i \implies z^6 =$ $\boxed{-8i}$

- Dati W e Z i seguenti sottospazi di $\mathbb{R}^3$:

$$W = \{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \ : \ x_1 + 3x_2 - x_3 = 0\}\ ,\ Z = \langle \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \rangle .$$ Allora $\mathbb{R}^3 = W \bigoplus Z$ | vero | falso |

- Determinare una base di $W \cap Z$

$$\boxed{\left\{ \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \right\}}$$

- $A = \begin{pmatrix} 1 & 0 & 0 & 1 & 2 & 2 & 2 \\ 2 & 0 & 0 & 2 & 4 & 4 & 4 \\ 2 & 0 & 0 & 2 & 4 & 4 & 4 \end{pmatrix} \implies \dim(Ker(l_A)) =$ $\boxed{6}$ $\quad \mathrm{rg}(A) =$ $\boxed{1}$

- $\det \begin{pmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 2 & 1 & -1 & 1 \\ 1 & -3 & 1 & 3 \end{pmatrix} =$ $\boxed{11}$

- $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \implies m.g.(1) =$ $\boxed{2}$

- $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix} \implies A \cdot B^t =$ $\begin{pmatrix} -1 & 4 \\ -2 & 3 \end{pmatrix}$

- Sia $f : \mathbb{R}^2 \to \mathbb{R}^2$ lineare. Sapendo che $f\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $f\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, allora $f\begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

28-1-2020

(1)

Traccia sol

$$\begin{cases} z^2 = -i\,|z|\cdot\bar{z} \\ |e^z| = e^{\sqrt{3}} \end{cases}$$

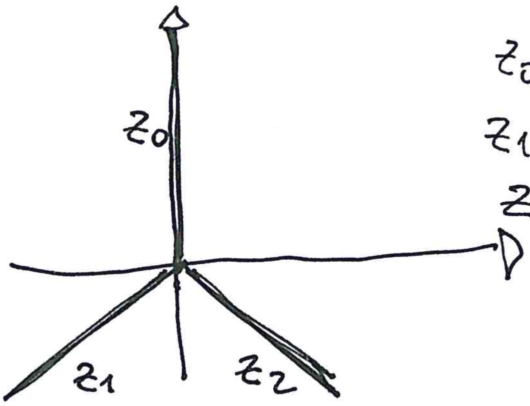
1. eq. $\quad z = \rho e^{i\vartheta} \Rightarrow \rho^2 \cdot e^{i2\vartheta} = e^{i\frac{3}{2}\pi}\cdot\rho\cdot\rho\cdot e^{-i\vartheta}$

$|z| = \rho$

$\bar{z} = \rho\cdot e^{-i\vartheta}$

$$\Leftrightarrow \begin{cases} \rho^2 = \rho^2 \\ 2\vartheta = \frac{3}{2}\pi - \vartheta + 2k\pi \quad , k \in \mathbb{Z} \end{cases}$$

sol. distinte $\quad \begin{cases} \rho \in \mathbb{R}^+ \\ \vartheta = \frac{\pi}{2} + \frac{2k\pi}{3} \quad k = 0,1,2 \end{cases}$



$z_0 = \rho\cdot i$

$z_1 = \rho\left(-\frac{\sqrt{3}}{2} - i\,\frac{1}{2}\right)$

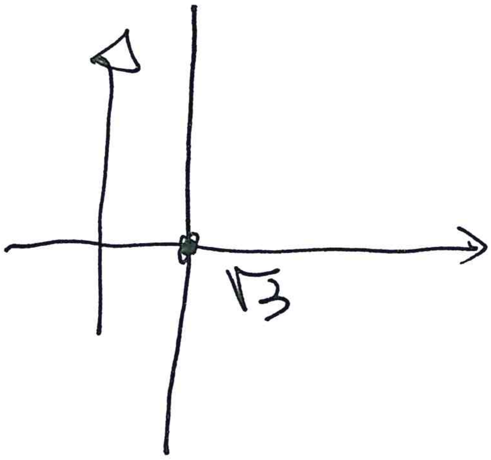
$z_2 = \rho\left(+\frac{\sqrt{3}}{2} - i\,\frac{1}{2}\right)$

2 eq.

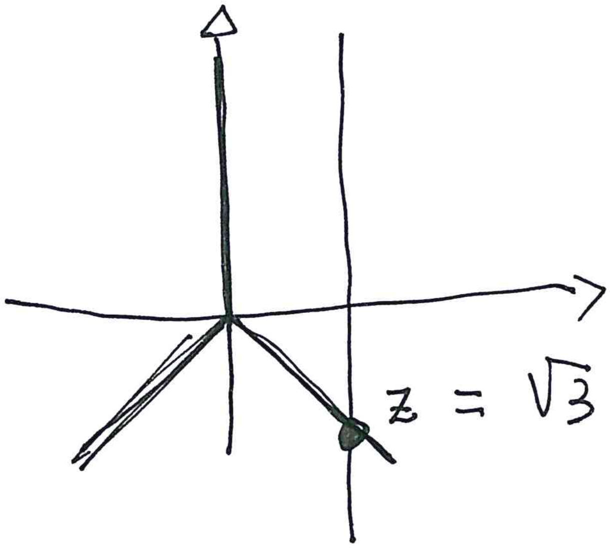
$$|e^z| = |e^{x+iy}| = |e^x| \cdot \underbrace{|e^{iy}|}_{=1} = e^{\sqrt{3}}$$

$$\iff \quad e^x = e^{\sqrt{3}}$$

$$\iff \quad \begin{cases} x = \sqrt{3} \\ y \text{ qualsiasi} \end{cases}$$



SOL. SISTEMA



$z = \sqrt{3} - i$

cerchiamo $\rho \in \mathbb{R}^+$ t.c. $\mathrm{Re}(\rho \cdot e^{i\theta}) = \sqrt{3}$

$\mathrm{Re}(\rho \cdot i) = \sqrt{3}$ impossibile

$\mathrm{Re}\left(\rho\left(-\frac{\sqrt{3}}{2} - i\frac{1}{2}\right)\right) = \sqrt{3}$ impossibile

$\mathrm{Re}\left(\rho \cdot \left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right)\right) = \sqrt{3} \implies \rho = 2$

SOLUZIONE: $z = \sqrt{3} - i$

2) $A_t = \begin{pmatrix} t & 0 & -1 \\ 2 & t & 1 \\ 0 & 1 & 3 \end{pmatrix}$

i) $\det = t^2 - t - 2$

$\det = 0 \iff t = 2, -1$

$t \neq 2, -1 \quad \begin{cases} rg = 3 \\ \dim(\ker) = 0 \end{cases}$

$M = \begin{pmatrix} 2 & t \\ 0 & 1 \end{pmatrix} \quad \det \neq 0 \ \forall t$

$\Rightarrow \quad t = 2, -1 \quad \begin{cases} rg = 2 \\ \dim(\ker) = 1 \end{cases}$

ii) $t \neq 2, 1 \qquad rg(A) = rg(A|b) = 3$

$\Rightarrow \exists !$ SOL.

$t = -1 \quad rg(A) = 2 < 3 = rg(A|b)$

$\Rightarrow$ non $\exists$ SOL.

$t = 2 \qquad b = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$

$\Rightarrow rg(A) = rg(A|b) = 2$

$\Rightarrow \exists \infty$ SOLUZIONI

iii) $t=2$ $\quad A = \begin{pmatrix} 2 & 0 & -1 \\ 2 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

$rg(A)=2 \quad v_1, v_2$ lin. ind.

$\Downarrow$

$$Im(L_A) = \left\langle \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \right\rangle$$

dim $=2 \quad \Rightarrow \quad 3-2=1$ equazioni

$$a_1 x_1 + a_2 x_2 + a_3 x_3 = 0$$

Imponiamo condizioni : $v_1 \in Im(L_A)$
$v_2 \in Im(L_A)$

$$\begin{cases} 2a_1 + 2a_2 = 0 \\ \quad\quad 2a_2 + a_3 = 0 \end{cases}$$

SOLUZIONE:

$$-x_1 + x_2 - 2x_3 = 0$$

3 $f: \mathbb{R}^3 \to \mathbb{R}^3$

$$Im(f) = \left\langle \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\rangle$$

$$Ker(f) = \left\langle \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

$f \Leftrightarrow A$ matrice 3×3

$$rg = 1 \to A = \begin{pmatrix} s & t & u \\ -2s & -2t & -2u \\ s & t & u \end{pmatrix}$$

Per semplicità poniamo $s = 1$

$$A \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow u = 0$$

$$A \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 1 & t & 0 \\ -2 & -2t & 0 \\ 1 & t & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow t = -1$$

$$A = \begin{pmatrix} 1 & -1 & 0 \\ -2 & 2 & 0 \\ 1 & -1 & 0 \end{pmatrix}$$

2$^{\text{ndo}}$ metodo

$$\operatorname{Ker}(f) = \left\langle \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

$$\Updownarrow$$

$$\left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} : \quad x_1 - x_2 = 0 \right\}$$

Quindi: righe di A multiple di $(1 \ \ -1 \ \ 0)$

colonne di A multiple di $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

$$\Rightarrow \quad A = \begin{pmatrix} 1 & -1 & 0 \\ -2 & 2 & 0 \\ 1 & -1 & 0 \end{pmatrix}$$

4

$$A = \begin{pmatrix} -1 & 0 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$p_A(\lambda) = \lambda^2$$

$\lambda = 0 \quad m.a. = 4$

$m.g.(0) = 4 - rg(A) = 2$

A è triang.le

A non è diag.le

AUTOSPAZIO

$$V_0 = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\rangle$$