Corso di laurea in Ingegneria Gestionale/ Chimica
Esame di ALGEBRA LINEARE - anno accademico 2013/2014

Prova scritta del 13/01/2014
TEMPO A DISPOSIZIONE: 120 minuti

(Cognome)	MARCO (Nome)	(Numero di matricola)

PRIMA PARTE

PUNTEGGIO : risposta mancante = 0 ; risposta esatta = +1 risposta sbagliata = -1
calcoli e spiegazioni non sono richiesti

- Sia $z = \sqrt{3} - \sqrt{3}i$.

  (i) Scrivere z nella rappresentazione trigonometrica $z = \varrho \cdot e^{i\vartheta}$: $z = \boxed{\sqrt{6} \cdot e^{i\frac{7}{4}\pi}}$

  (ii) $z^4 = \boxed{-36}$

- Dati W e Z i seguenti sottospazi di $\mathbb{R}^3$:

  $$W = \{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \ : \ x_1 + x_2 - x_3 = 0 \} ; \qquad Z = \langle \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \rangle .$$

  Allora:

  $\dim(W) = \boxed{2}$ $\dim(W \cap Z) = \boxed{1}$ $\dim(W + Z) = \boxed{3}$

- $A = \begin{pmatrix} 1 & 1 & 1 & 5 & 5 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \implies \dim(Ker(l_A)) = \boxed{3}$ $\quad rg(A) = \boxed{2}$

- $\det \begin{pmatrix} 0 & 1 & 0 & 2 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 1 & 3 & 0 & 0 \end{pmatrix} = \boxed{-3}$

- Data $A = \begin{pmatrix} 2 & 3 & 2 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ si consideri l'autovalore $\lambda_0 = 2$. Allora: $m.g.(2) = \boxed{1}$

- $A = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 0 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 1 & 1 \end{pmatrix} \implies A \cdot B =$ $\begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$

compito 13-1-2014

Treccia sol

(1)
$$\begin{cases} (z-2i)^4 = -4 \\ |e^{iz}| = \dfrac{1}{e} \end{cases}$$

i) Poniamo $w = z - 2i$

$$w^4 = \rho^4 \cdot e^{i4\vartheta} = 4 \cdot e^{i\pi} = -4$$

$$\Leftrightarrow \begin{cases} \rho^4 = 4 & , \rho \in \mathbb{R}^+ \\ 4\vartheta = \pi + 2k\pi & , k \in \mathbb{Z} \end{cases}$$

Sol. distinte
$$\begin{cases} \rho = \sqrt[4]{4} = \sqrt{2} \\ \vartheta = \dfrac{\pi + 2k\pi}{4} & , k = 0, \dots, 3 \end{cases}$$

$$w_i \begin{cases} 1 + i \\ -1 + i \\ -1 - i \\ 1 - i \end{cases}$$

$\boxed{z = w + 2i}$ $\Longrightarrow$

$z_0 = 1 + 3i$

$z_1 = -1 + 3i$

$z_2 = -1 + i$

$z_3 = 1 + i$

ii) $|e^{iz}| = |e^{ix-y}| =$ ②

$= \underbrace{|e^{ix}|}_{=1} \cdot |e^{-y}| = e^{-y}$

CIOE' $|e^{iz}| = \frac{1}{e}$

$\Leftrightarrow \begin{cases} e^{-y} = \frac{1}{e} \leftrightarrow y = 1 \\ x \text{ qualsiasi} \end{cases}$

CONCLUSIONE: $z_2 = -1+i$

$z_3 = 1+i$

2

$$A_t = \begin{pmatrix} 4 & 4 & 4 \\ t & 1 & 1 \\ 1 & t & 1 \end{pmatrix}$$

$$\det(A_t) = 4t^2 - 8t + 4$$
$$= 4 \cdot (t-1)^2$$

$$\det(A_t) = 0 \iff t = 1$$

- $t \neq 1$ $\quad \begin{cases} rg(A) = 3 \\ \dim(\ker(f_A)) = 0 \end{cases}$

- $t = 1$: $\quad A = \begin{pmatrix} 4 & 4 & 4 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$rg(A) = 1$ (3 colonne uguali)

$\dim(\ker(f_A)) = 3 - 1 = 2$

ii) A_t 3×3

$(A_t \vdots b)$ 3×4

Se $rg(A_t)=3 \Rightarrow \exists!$ sol.

$\Updownarrow$

$t\neq 1$

$t=1$: $rg(A)=1$

$$(A\vdots b)=\begin{pmatrix}4&4&4&1\\1&1&1&0\\1&1&1&2\end{pmatrix}$$

$rg(A\vdots b)=2$

Quindi $rg(A\vdots b)=2>1=rg(A)$

$\Rightarrow$ non $\exists$ sol.

iii) $W = \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$ $\dim(W) = 1$ (5)

$$\mathbb{R}^3 = W \oplus \operatorname{Ker}(\varrho_A)$$

$$\Updownarrow$$

$$\begin{cases} \dim \operatorname{Ker}(\varrho_A) = 2 \\ \operatorname{Ker}(\varrho_A) \cap W = \{0_V\} \end{cases}$$

unico caso possibile : $t=1$

$\operatorname{Ker} \leftrightarrow \{ 4x_1 + 4x_2 + 4x_3 = 0 \}$

$$W = \left\{ t \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \mid t \in \mathbb{R} \right\}$$

$$W : \begin{cases} x_1 = t \\ x_2 = t \\ x_3 = t \end{cases}$$

$W \cap \operatorname{Ker} :$ $4t + 4t + 4t = 0$

$$\Leftrightarrow \quad t = 0$$

CIOÈ' $W \cap \operatorname{Ker}(\varrho_A) = \{0_V\}$.

$t=1$ $\boxed{\exists \mathbb{R}^3 = W \oplus \operatorname{Ker}(\varrho_A)}$ O.K.

3 $\mathrm{Im}(f) = \left\langle \begin{pmatrix} 5 \\ 1 \\ 1 \end{pmatrix} \right\rangle$

$\mathrm{Ker}(f) = \left\langle \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right\rangle$

$f \Longleftrightarrow A$ matrice 3×3

$\mathrm{rg}(A) = 1$

Possiamo scegliere $A = \begin{pmatrix} 5 & 5\alpha & 5\beta \\ 1 & \alpha & \beta \\ 1 & \alpha & \beta \end{pmatrix}$

$\mathrm{Ker}(f) = \left\langle \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right\rangle$

$\Longleftrightarrow \begin{cases} A \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\ A \cdot \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \end{cases}$

$\Longleftrightarrow \begin{cases} 1 + \alpha + 2\beta = 0 \\ 2 + \beta = 0 \end{cases}$

sol: $\beta = -2$, $\alpha = 3$

$A = \begin{pmatrix} 5 & 15 & -10 \\ 1 & 3 & -2 \\ 1 & 3 & -2 \end{pmatrix}$

4

$$A = \begin{pmatrix} 3 & 2 & 0 & 3 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

$$P_A(\lambda) = \dots = \lambda^2 \cdot (\lambda - 4)^2$$

AUTOVALORI: $\quad 0 \quad m.a. = 2$

$\qquad\qquad\qquad\quad 4 \quad m.a. = 2$

$$m.g.(0) = \dim \ker (\varrho_A)$$

$$rg(A) = 2 \Rightarrow \dim \ker (\varrho_A) = 4 - 2$$

$$\Rightarrow m.g.(0) = 2$$

$$m.g.(0) = \dim \ker (\varrho_A - 4\, id)$$

$$rg(A - 4\, Id) = rg \begin{pmatrix} -1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 \\ 1 & 1 & 0 & -3 \end{pmatrix} = 3$$

$$\Rightarrow m.g.(4) = 4 - 3 = 1$$

Autovettori relativi a $\lambda = 0$

$\Leftrightarrow \operatorname{Ker}(\ell_A)$

$$\Leftrightarrow \begin{cases} 3x_1 + 2x_2 + 3x_4 = 0 \\ 4x_2 = 0 \\ x_1 + x_2 + x_4 = 0 \end{cases}$$

$$\text{Sol} : \left\{ s \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} : (s,t) \neq (0,0) \right\}$$

Autovettori relativi a $\lambda_1 = 4$

$\Leftrightarrow \operatorname{Ker}(\ell_A - 4\,id)$

$$\Leftrightarrow \begin{cases} -x_1 + 2x_2 + 3x_4 = 0 \\ -x_3 = 0 \\ x_1 + x_2 - 3x_4 = 0 \end{cases}$$

$$\text{sol.} = \left\{ s \cdot \begin{pmatrix} 3 \\ 0 \\ 0 \\ 1 \end{pmatrix} : s \neq 0 \right\}$$

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iii) A è triang.

A non è diag. perché $mg(4) \neq ma(4)$