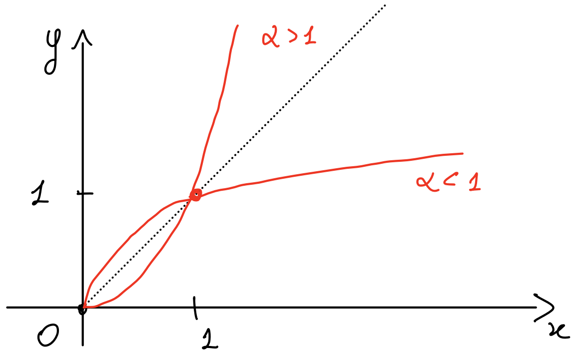
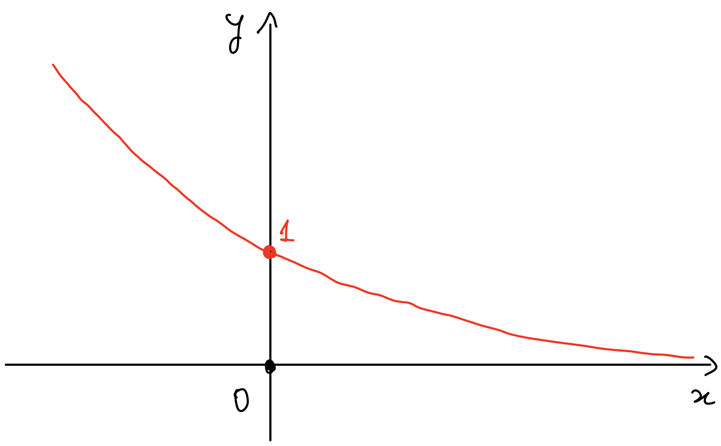
Grafici di funzioni elementari $\quad f:\mathbb{R}\to\mathbb{R}$

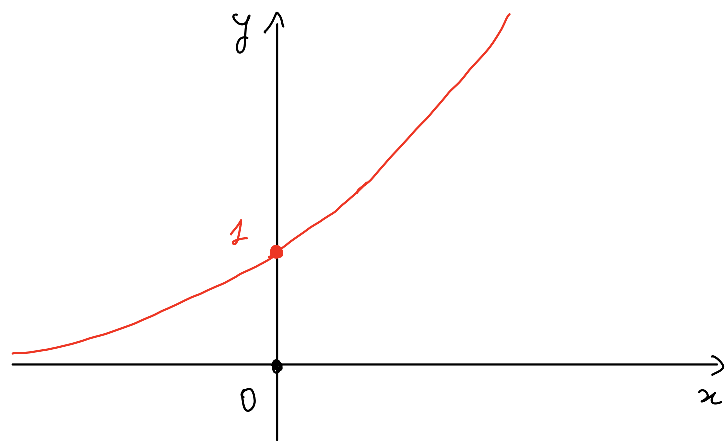
- $f(x)=x^n$, $\quad n\in\mathbb{N}$
- $f(x)=x^{\frac{m}{n}}$, $\quad m\in\mathbb{Z}$, $n\in\mathbb{N}$, $m\neq 0$. $\quad D=\mathbb{R}$ op. $[0,+\infty)$?
- $f(x)=x^{\alpha}$, $\quad \alpha\in(0,+\infty)$. $\quad D=[0,+\infty)$, $\quad 0^{\alpha}=0$, $\forall\alpha\neq 0$.



- Esponenziale. $\quad f(x)=a^x$, $\quad a\in(0,+\infty)$, $a\neq 1$, $\quad e=2.7\ldots$

$a<1 \qquad a^{-|x|}=\left(\frac{1}{a}\right)^{|x|} \qquad a>1 \ (a=e)$





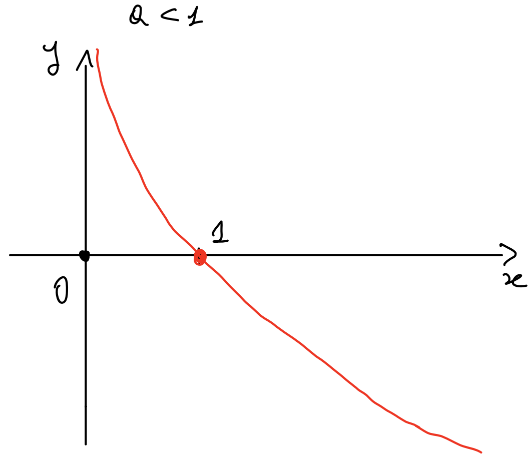
$$\operatorname{Im} f=(0,+\infty)$$

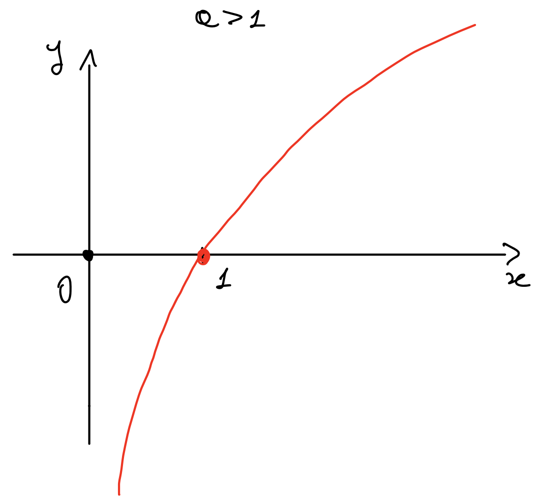
$\operatorname{Im} f=(0,+\infty) \iff \forall y\in(0,+\infty)\ \exists\, x\in\mathbb{R}$ t.c. $y=f(x)$

$\iff \forall y\in(0,+\infty)\ \exists\, x\in\mathbb{R}$ t.c. $y=a^x$

$$y=a^x \iff x=\log_a y \quad \forall a\neq 1.$$

- $f(x)=\log_a x$, $\quad a\in(0,+\infty)$, $a\neq 1$. $\quad D=(0,+\infty)$





$$\mathrm{Im}\, f = \mathbb{R}$$

Esercizi

Determinare dominio naturale e immagine di

- $f(x) = e^{2x+1}$

$$D = \{ x \in \mathbb{R} \,/\, 2x+1 \in \mathbb{R} = \text{dominio naturale della funz. esp.} \} = \mathbb{R}$$

$$\mathrm{Im}\, f = \{ y \in \mathbb{R} \,/\, \exists\, x \in D = \mathbb{R} \text{ tale che } y = e^{2x+1} \} = (0, +\infty)$$

$$y = e^{2x+1} \iff \log_e y = 2x+1 \iff x = \frac{\log_e y - 1}{2}$$

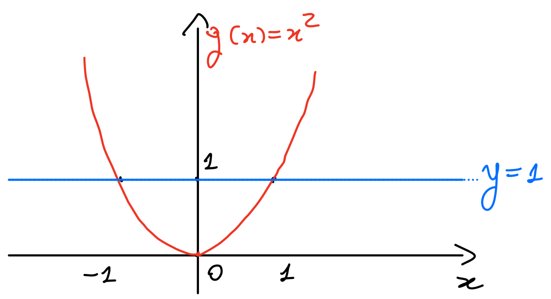
$$\Downarrow$$

$$y \in (0, +\infty)$$

- $f(x) = \log_e (x^2 - 1)$

$$D = \{ x \in \mathbb{R} \,/\, x^2 - 1 \in \underbrace{(0, +\infty)}_{\text{dominio naturale di } \log_e} \} = (-\infty, -1) \cup (1, +\infty)$$

$$x^2 - 1 > 0 \iff x^2 > 1 \iff \sqrt{x^2} > \sqrt{1} \iff |x| > 1$$



$$\{ x \in \mathbb{R} \,/\, x^2 > 1 \} \iff \{ x \in \mathbb{R} \,/\, g(x) > 1 \}$$

$$\mathrm{Im}\, f = \{ y \in \mathbb{R} \,/\, \exists\, x \in (-\infty, -1) \cup (1, +\infty) \text{ tale che } y = \log_e (x^2 - 1) \}$$

$$y = \log_e (x^2 - 1) \iff e^y = e^{\log_e (x^2-1)} \iff e^y = x^2 - 1 \iff x^2 = e^y + 1$$

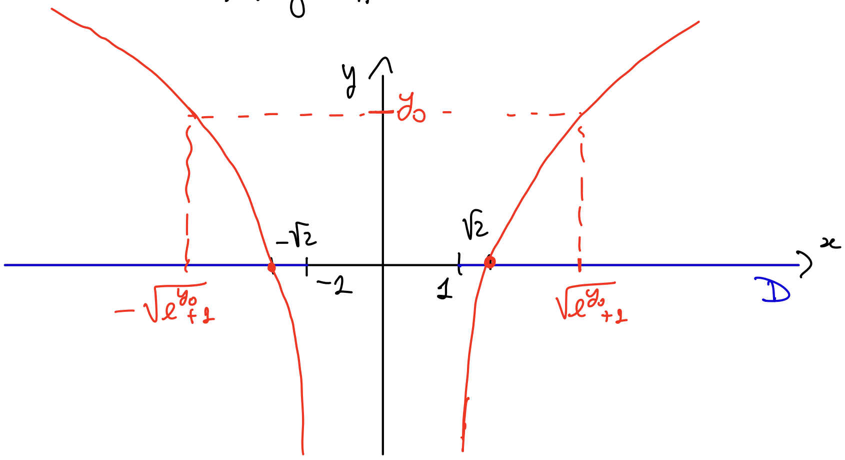
$\Longleftrightarrow \quad |x| = \sqrt{e^y+1} \qquad \text{per} \quad x \in (-\infty,-1) \cup (1,+\infty)$

$e^y > 0 \ \forall y \in \mathbb{R} \iff e^y+1 > 1 \ \forall y \in \mathbb{R} \iff \sqrt{e^y+1} > 1 \ \forall y \in \mathbb{R}$

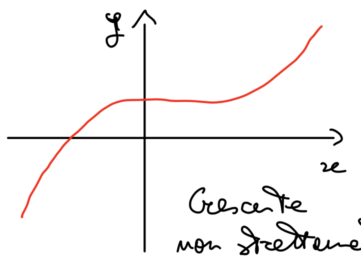
Quindi $\forall y \in \mathbb{R} \quad |x| = \sqrt{e^y+1} > 1 \iff x \in (-\infty,-1) \cup (1+\infty)$

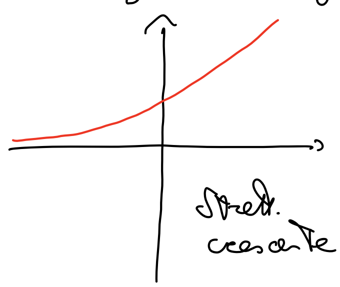
e $\quad y = \log_e(x^2-1)$

$\Rightarrow \operatorname{Im} f = \mathbb{R}.$

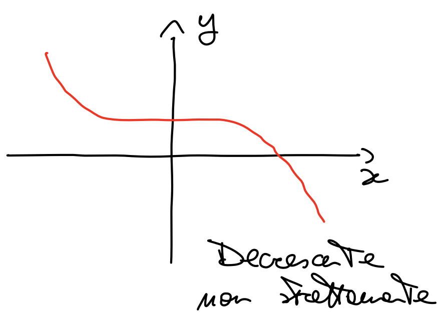


Def $f: \mathbb{R} \to \mathbb{R}$, D dominio naturale. Si dice MONOTONA (STRETTAMENTE) CRESCENTE se $\forall x_1, x_2 \in D$ con $x_1 < x_2$ si ha $f(x_1) \le f(x_2)$ $\left(f(x_1) < f(x_2) \right)$

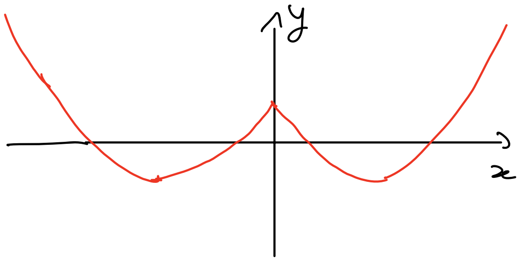




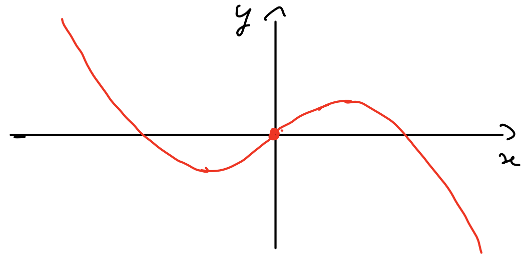
Si dice MONOTONA (STRETTAMENTE) DECRESCENTE se $\forall x_1, x_2 \in D$ con $x_1 < x_2$ si ha $f(x_1) \ge f(x_2)$ $\left(f(x_1) > f(x_2) \right)$.



Def $f: \mathbb{R} \to \mathbb{R}$, D dominio naturale tale che se $x \in D$ allora $-x \in D$.
Si dice che f è PARI se $f(x) = f(-x) \ \forall x \in D$ $\left(f(x) = \log_e(x^2-1) \right)$
Si dice che f è DISPARI se $f(-x) = -f(x) \ \forall x \in D$ $\left(f(x) = x^3 \right)$

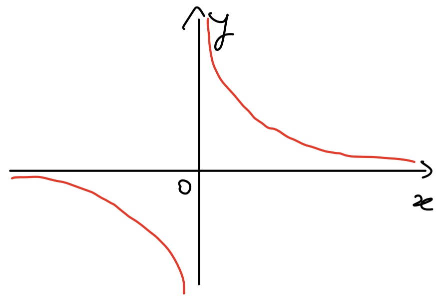


PARI



DISPARI

Es

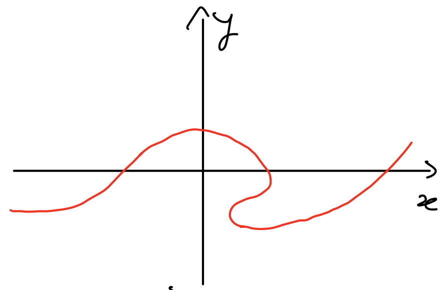


$D = \mathbb{R} \setminus \{0\}$

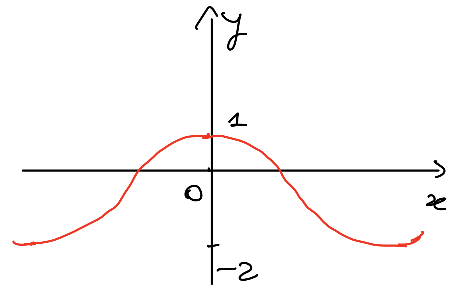
$\mathrm{Im}\, f = \mathbb{R} \setminus \{0\}$

Dispari

(Es $f(x) = \frac{1}{x}$)



NON È GRAFICO DI FUNZIONE



$D = \mathbb{R}$

$\mathrm{Im}\, f = [-2, 1]$

Pari

(Es $f(x) = \frac{3}{2} \cos x - \frac{1}{2}$)

Funzioni Trigonometriche

$\alpha = 35^\circ$

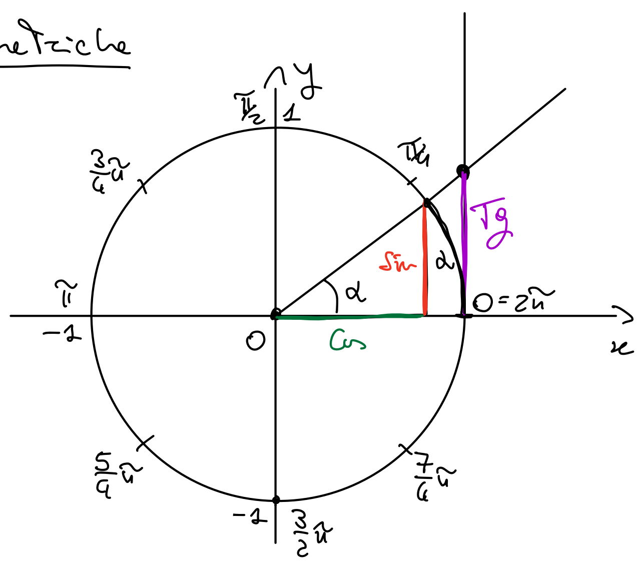
$35 : 360 = \alpha : 2\pi$

$\alpha = \frac{35^\circ \cdot 2\pi}{360}$

$45^\circ \longleftrightarrow \frac{\pi}{4}$ Rad

$90^\circ \longleftrightarrow \frac{\pi}{2}$ Rad

$180^\circ \longleftrightarrow \pi$ Rad



$\alpha \in [0, 2\pi)$

$\sin \alpha \in [-1, 1]$

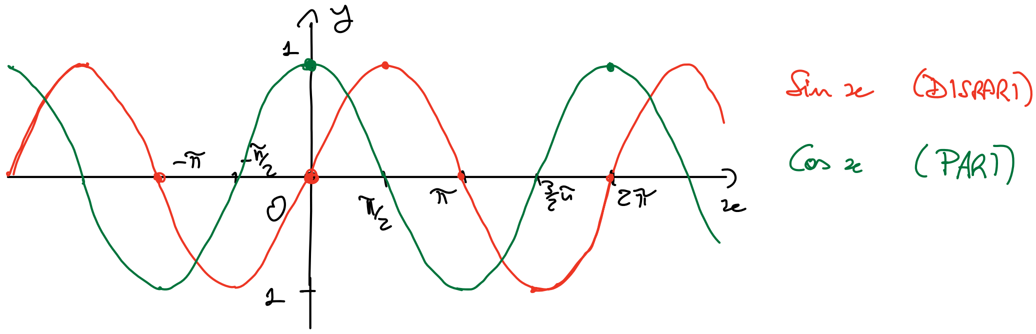
$\cos \alpha \in [-1, 1]$

$\mathrm{Tg}\, \alpha \in \mathbb{R}$

$\nexists\ \mathrm{Tg}\, \frac{\pi}{2},\ \mathrm{Tg}\, \frac{3}{2}\pi$

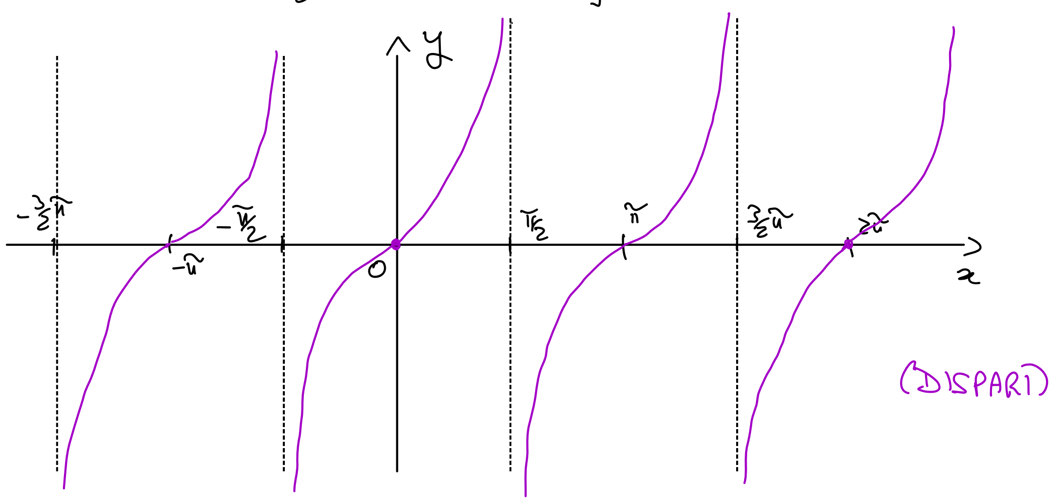
- $\sin^2 \alpha + \cos^2 \alpha = 1$
- $\alpha \in [0, \frac{\pi}{2}]$, $\quad \sin \alpha \leq \alpha \leq \mathrm{Tg}\, \alpha$

Def La funzione $\sin x : \mathbb{R} \to \mathbb{R}$, $\cos x : \mathbb{R} \to \mathbb{R}$ ponendo $\sin x = \sin(x + 2\pi)$, $\cos x = \cos(x + 2\pi)$ $\forall x \in \mathbb{R}$.



La funzione $Tg\, x : \mathbb{R} \to \mathbb{R}$ ponendo $Tg\, x = Tg(x+\pi) \quad \forall x \in D$

$$D = \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi \;/\; k \in \mathbb{Z} \right\}$$



<u>ESERCIZIO</u> Trovare dominio naturale, immagine e grafico di

$$f(x) = \log_e\left(|x^2-1|\right)$$