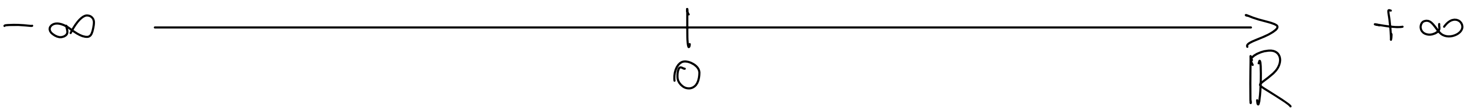
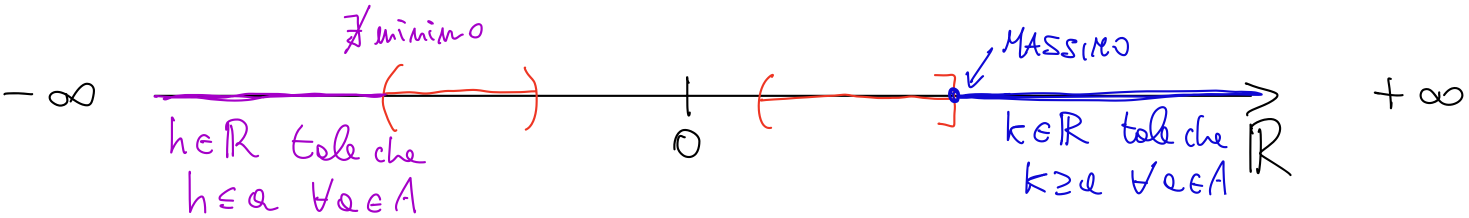


Insieme A $\to$ maggioranti, minoranti, limitato, massimo e minimo



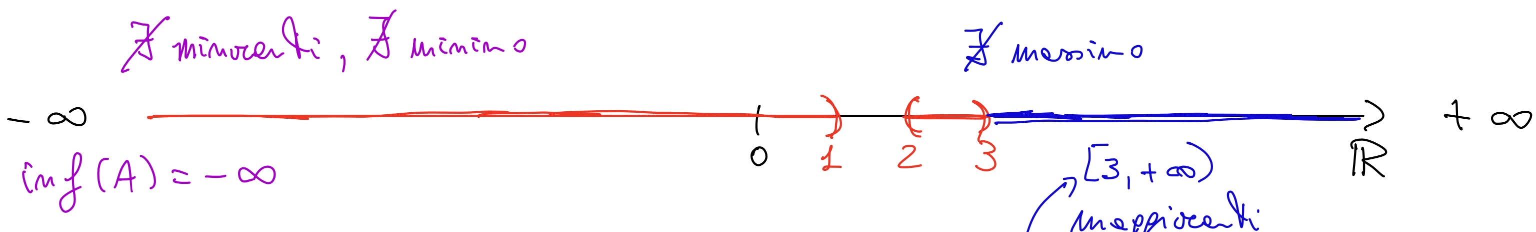
A è limitato sup. se $\exists$ maggioranti
" inf se $\exists$ minoranti

Definizione Dato $A \subseteq \mathbb{R}$, si chiama ESTREMO SUPERIORE DI A

$$\sup(A) := \begin{cases} \text{il minimo dei maggioranti} & \text{se } A \text{ è limitato superiormente} \\ +\infty & \text{se } A \text{ non è limitato superiormente} \end{cases}$$

Si chiama ESTREMO INFERIORE DI A

$$\inf(A) := \begin{cases} \text{il massimo dei minoranti} & \text{se } A \text{ è limitato inferiormente} \\ -\infty & \text{se } A \text{ non è limitato inferiormente} \end{cases}$$

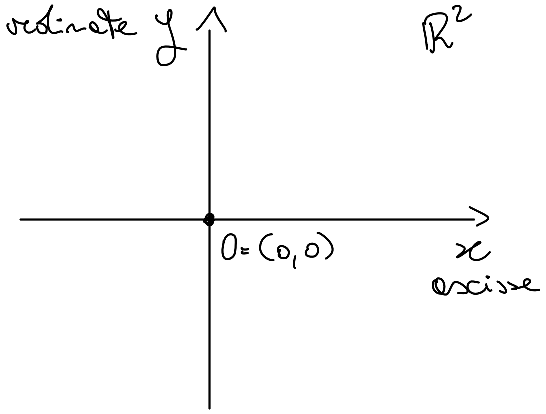


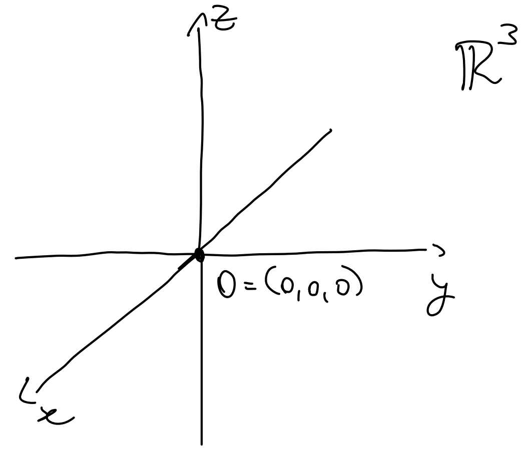
$A = (-\infty, 1) \cup (2, 3)$

$\sup(A) = 3$

Piano Cartesiano $\mathbb{R}^2 := \mathbb{R} \times \mathbb{R} = \{(x, y) \,/\, x \in \mathbb{R},\ y \in \mathbb{R}\}$

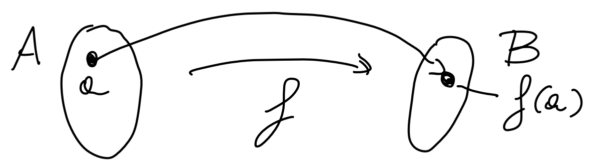
Spazio Cartesiano $\mathbb{R}^3 := \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x, y, z) \,/\, x \in \mathbb{R},\ y \in \mathbb{R},\ z \in \mathbb{R}\}.$





Funzioni

Def Dati due insiemi A, B, una funzione $f : A \to B$ è una legge che associa ad ogni elemento di A uno ed un solo elemento di B.



$A \ni a \longmapsto f(a) \in B$.

A dominio di f, B codominio di f,

Immagine di $f = f(A) := \{ b \in B \,/\, \exists\, a \in A$ tale che $f(a) = b \} \subseteq B$.

$$f(A) = \bigcup_{a \in A} f(a)$$

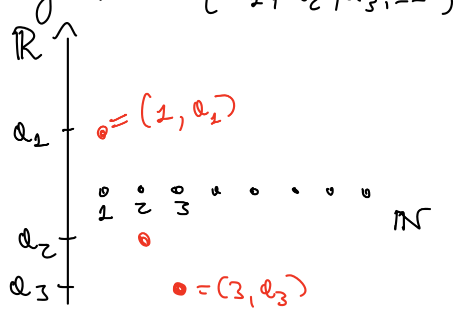
Grafico di $f := \{ (a,b) \in A \times B \,/\, b = f(a) \} \subset A \times B$

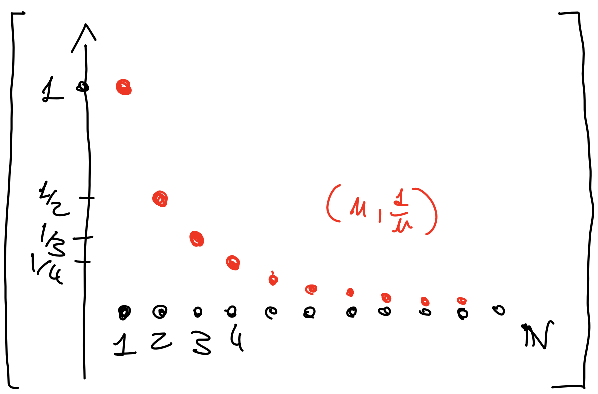
Esempio Successioni $\{a_n\}_{n \in \mathbb{N}} \subset \mathbb{R}$ $\left[\ \{\frac{1}{n}\}_{n \in \mathbb{N}} = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\} \subset \mathbb{R} \right]$

$f : \mathbb{N} \to \mathbb{R}$, $\mathbb{N}$ dominio, $\mathbb{R}$ codominio

$f(n) = a_n \in \mathbb{R}$ $\qquad \left[f(n) = \frac{1}{n} \in \mathbb{R} \right]$

$f(\mathbb{N}) = \{a_1, a_2, a_3, \dots\}$ $\qquad \left[f(\mathbb{N}) = \{1, \frac{1}{2}, \frac{1}{3}, \dots\} \right]$





Graf $f = \{ (n, y) \in \mathbb{N} \times \mathbb{R} \,/\, y = a_n \}$

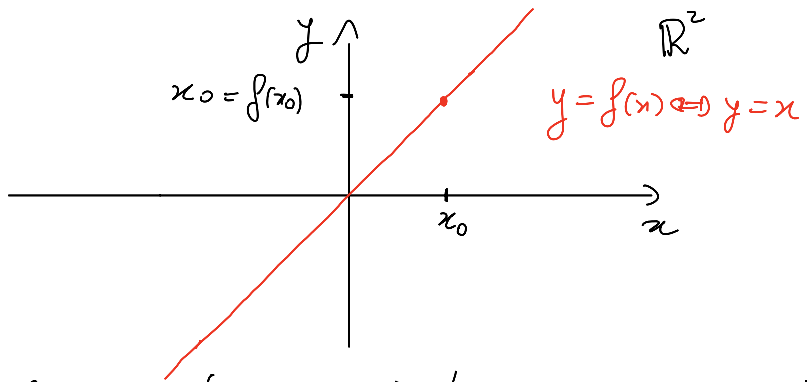
Funzioni reali di variabile reale $f: \mathbb{R} \to \mathbb{R}$

<u>Def</u> Data una $f: \mathbb{R} \to \mathbb{R}$ di legge $f(x)$, si chiama DOMINIO NATURALE l'insieme $D \subseteq \mathbb{R}$ (dominio) per cui $f(x)$ ha senso $\forall x \in D$.

<u>Esempi</u>

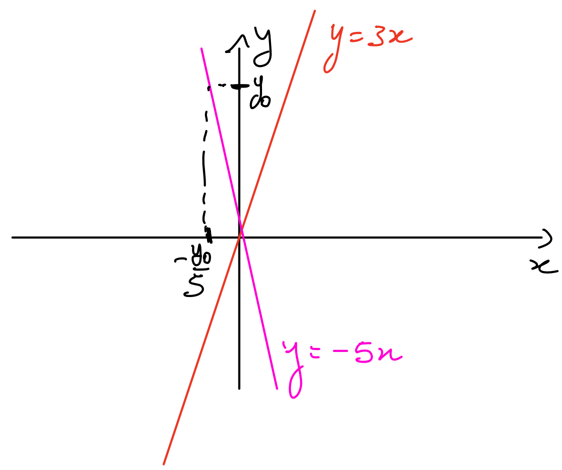
- Polinomi, $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$, $a_i \in \mathbb{R}$, $n \in \mathbb{N}$

- $f(x) = x$, $D = \mathbb{R}$, $f(\mathbb{R}) = \text{Imm}(f) = \mathbb{R} \subseteq$ codominio



$$\text{Graf}(f) = \{ (x, y) \in \mathbb{R}^2 \ / \ x \in D,\ y = f(x) \}$$

- $f(x) = 3x$, $f(x) = -5x$.
  $\text{Imm} = \mathbb{R}$ $\qquad$ $\text{Imm} = \mathbb{R}$



$$\text{Imm} f = \mathbb{R} \iff \forall y \in \mathbb{R}\ \exists x \in D \text{ t.c. } y = f(x)$$

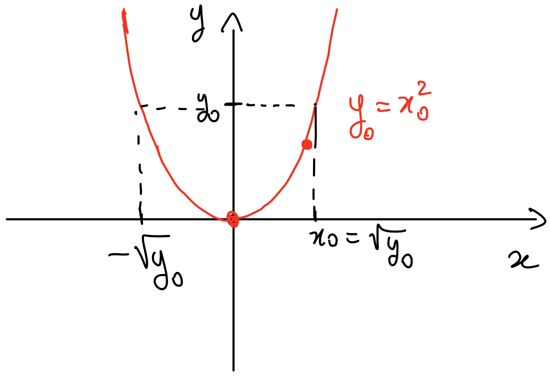
$f(x) = -5x$, $\text{Imm} f = \mathbb{R} \iff \forall y \in \mathbb{R}\ \exists x \in \mathbb{R}$ t.c. $y = -5x$

$$(2 = -5x \iff x = -\tfrac{2}{5})$$

$\text{Imm} f = \mathbb{R}$ perché $\forall y \in \mathbb{R}$, $f(\underbrace{-\tfrac{y}{5}}_{x}) = y$.

- $f(x) = x^2$, $D = \mathbb{R}$, $\text{Imm} f = [0, +\infty)$

$$\text{Graf} f = \{ (x, y) \in \mathbb{R}^2 \ / \ x \in \mathbb{R},\ y = x^2 \}$$

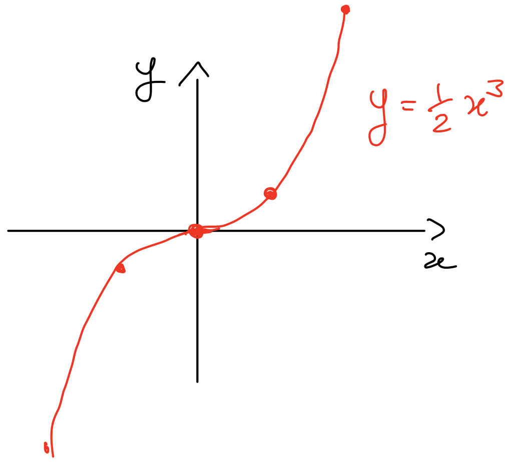


$\text{Imm } f = [0, +\infty) \iff \forall\, y \geq 0 \;\; \exists\, x \in \mathbb{R}$ t.c. $y = x^2$

$f(\sqrt{y}) = f(-\sqrt{y}) = y.$

<u>oss</u> $\sqrt{4} = 2$, $\sqrt{(-2)^2} = 2$, $\sqrt{x^2} = |x|$

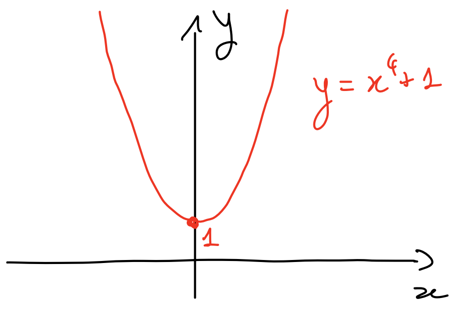
$- \; f(x) = \frac{1}{2} x^3$



$\forall y \in \mathbb{R}$

$f(\underbrace{\sqrt[3]{2y}}_{x}) = y$

$- \; f(x) = x^4 + 1$, $D = \mathbb{R}$, $\text{Imm } f = [1, +\infty)$



$y = x^4 + 1 \iff x = \pm (y-1)^{\frac{1}{4}}$

$- \; f(x) = |x|$, $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$, $D = \mathbb{R}$, $\text{Imm } f = [0, +\infty)$

$\forall y \geq 0$

$y = |x| \iff x = \pm y.$