Derivata $\Longleftrightarrow$ grafico di f.

$f: D \subseteq \mathbb{R} \to \mathbb{R}$, D dominio naturale,

$f': D' \subseteq \mathring{D} \to \mathbb{R}$, D' = punti di derivabilità

$\mathring{D}$ = punti interni di D

$x_0 \in D'$, $f'(x_0)$ è il coefficiente angolare della retta tangente al grafico di f nel punto $(x_0, f(x_0))$

retta: $y = \underset{\text{coeff. ang.}}{f'(x_0)}\,(x - x_0) + f(x_0)$

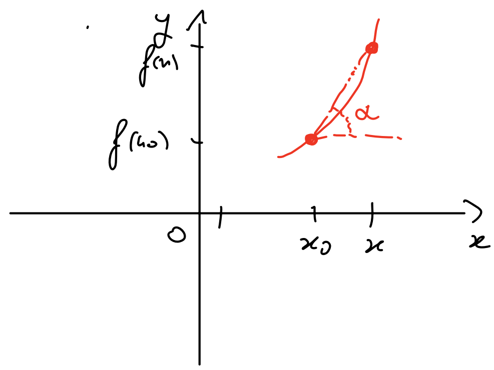
$\underbrace{f(x_0) - f'(x_0)x_0}_{}$ costante

$$\mathbb{R} \ni f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

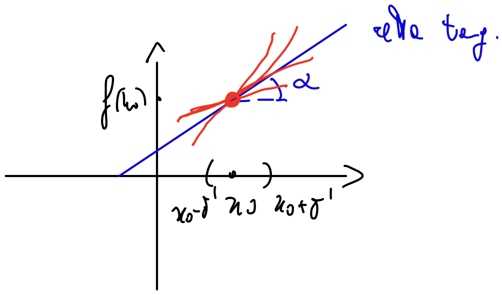
rapporto incrementale

$\shortparallel$

$\tan \alpha$



$x_0 \in D'$, $f'(x_0) > 0$



$f'(x_0) = \tan \alpha$

$$f'(x_0) > 0 \iff \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = f'(x_0) > 0 \iff$$

$\forall \varepsilon > 0$ $\exists \delta > 0$ tale che $\forall x \in (x_0 - \delta, x_0 + \delta) \setminus \{x_0\}$ si ha

t.c.

$f'(x_0) - \varepsilon > 0$ $\quad \dfrac{f(x) - f(x_0)}{x - x_0} \in (\underset{\underset{0}{\vee}}{f'(x_0) - \varepsilon}, f'(x_0) + \varepsilon)$

$$\Rightarrow \exists \delta' > 0 \text{ t.c. } \frac{f(x) - f(x_0)}{x - x_0} > 0 \quad \forall x \in (x_0 - \delta', x_0 + \delta')$$

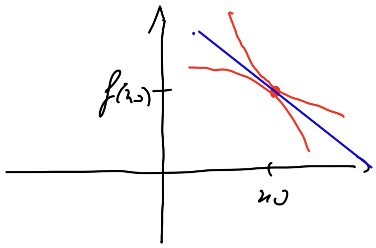
$x \in (x_0, x_0+\delta')$, $x > x_0 \Rightarrow f(x) > f(x_0)$
$x \in (x_0-\delta', x_0)$, $x < x_0 \Rightarrow f(x) < f(x_0)$

f è monotona crescente in $(x_0-\delta', x_0+\delta')$

$f'(x_0) < 0 \Rightarrow \exists\, \delta' > 0$ t.c.

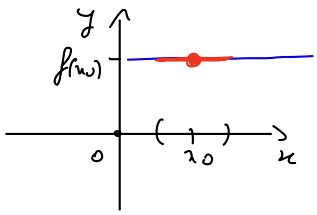
$x \in (x_0, x_0+\delta')$, $x > x_0 \Rightarrow f(x) < f(x_0)$
$x \in (x_0-\delta', x_0)$, $x < x_0 \Rightarrow f(x) > f(x_0)$

f è monotona decrescente in $(x_0-\delta', x_0+\delta')$



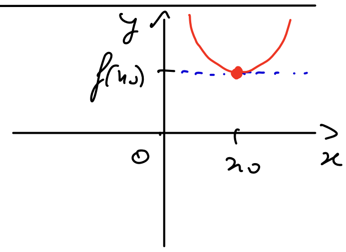
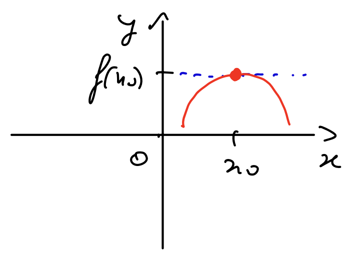
$f'(x_0) = 0$ $\quad x_0$ punto critico

- $\exists\, \delta > 0$ tale che $f'(x) = 0 \quad \forall x \in (x_0-\delta, x_0+\delta)$

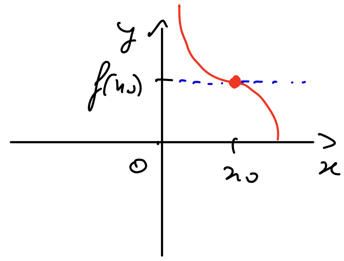


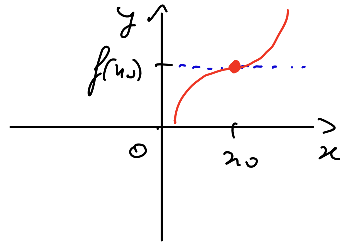
allora $f(x) = f(x_0) \quad \forall x \in (x_0-\delta, x_0+\delta)$

- $\exists\, \delta > 0$ tale che $f'(x) \neq 0 \quad \forall x \in (x_0-\delta, x_0+\delta)$

	$x \in (x_0-\delta, x_0)$	$x \in (x_0, x_0+\delta)$	
segno di $f'(x)$	< 0	> 0	 x_0 punto di minimo locale
	> 0	< 0	 x_0 punto di massimo locale

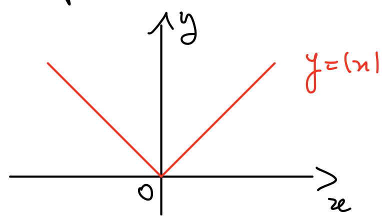
	< 0	< 0	
	> 0	> 0	x_0 punto di flesso





OSS x_0 punto di massimo locale / " " minimo locale $+ \ \exists f'(x_0) \Rightarrow f'(x_0) = 0$

ES $f(x) = |x|$ ha un punto di minimo in $x_0 = 0$, $f'(0)$ non esiste



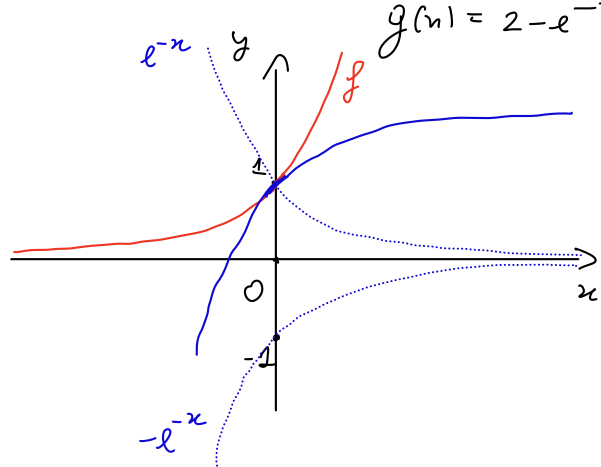
ES Determinare il numero di soluzioni di

$$e^x + e^{-x} = 2$$

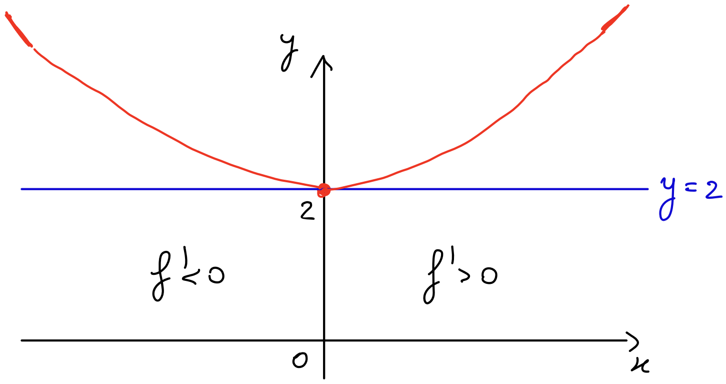
$e^x + e^{-x} = 2 \iff e^x = 2 - e^{-x}$

$f(x) = e^x$

$g(x) = 2 - e^{-x}$



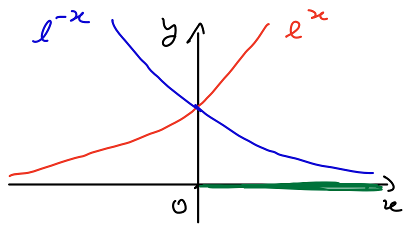
$f(x)=e^{x}+e^{-x}$, $D=\mathbb{R}$, $f(0)=e^{0}+e^{-0}=2$, f è pari $[f(-x)=f(x)]$



$$\#\{x : e^{x}+e^{-x}=2\} = 1$$

f è continua in $\mathbb{R}$, $\lim_{x\to+\infty} f(x) = +\infty + 0 = +\infty$, f è derivabile in $\mathbb{R}$

$$f'(x) = (e^{x})' + (e^{-x})' = e^{x} - e^{-x}$$

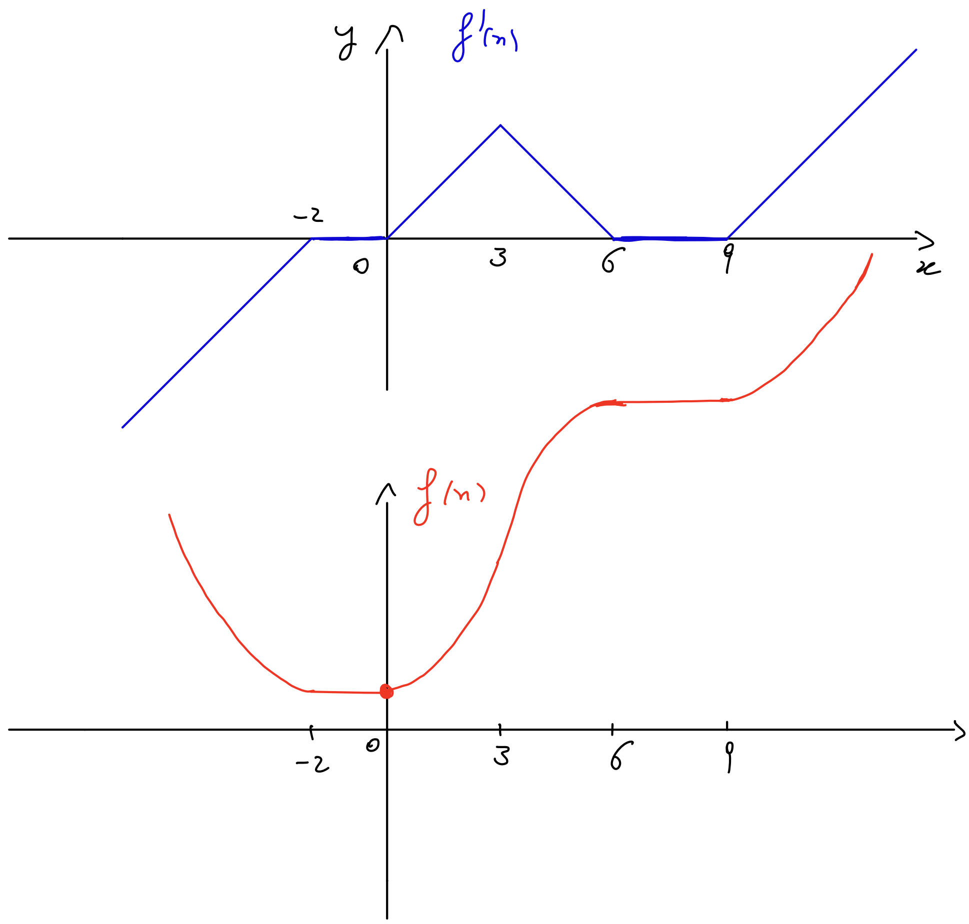


$$e^{x} - e^{-x} > 0 \iff \underline{e^{x}} > \underline{e^{-x}}$$

$$\iff x > 0$$

$$f'(x) \begin{cases} >0 & \iff x>0 \\ <0 & \iff x<0 \\ =0 & \iff x=0 \end{cases}$$

ES

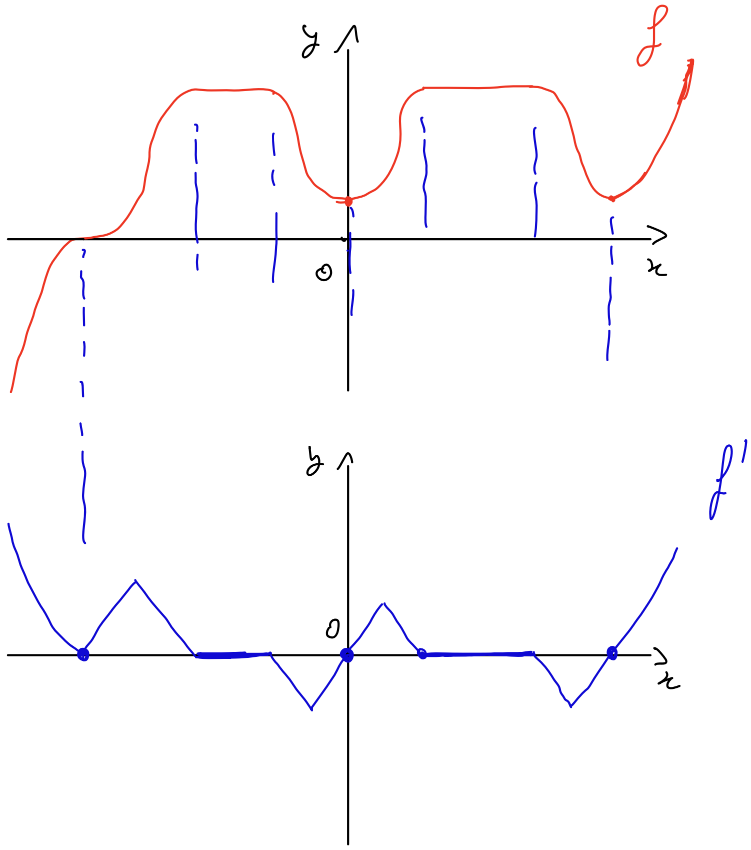


$(f(x)+c)' = f'(x)$

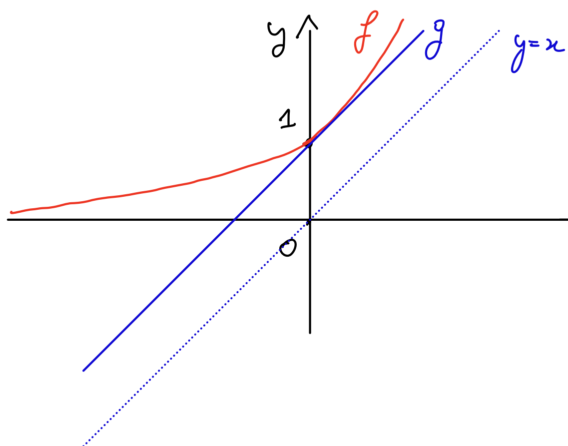
$\forall c \in \mathbb{R}$

$f(0)=1$

ES



ES Contiamo le soluzioni di $e^x = x + 1$



$f(x) = e^x$

$g(x) = x + 1$

$f(0) = 1$

$g(0) = 1$

$f(x)$, retta tangente al grafico di $f(x)$ in $x_0 = 0$? $\quad f'(x) = (e^x)' = e^x$

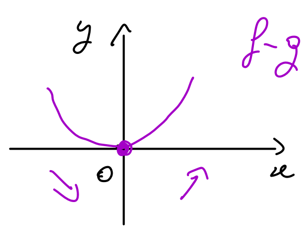
$$y = f'(x_0)(x - x_0) + f(x_0)$$

$$y = f'(0)\, x + f(0) \qquad f'(0) = e^0 = 1 \;,\; f(0) = e^0 = 1$$

$$y = x + 1$$

$f(x) - g(x) \geq 0$? $\quad f(0) - g(0) = 0$, $\quad (f(x) - g(x))' = f'(x) - g'(x) = e^x - 1$

$$(f(x) - g(x))' \quad \begin{array}{l} > 0 \iff e^x > 1 \iff x > 0 \\ < 0 \iff e^x < 1 \iff x < 0 \\ = 0 \iff e^x = 1 \iff x = 0 \end{array}$$



ES Trovare le soluzioni di $\log x = x - 1$

$$\left[\begin{aligned} (\log x)' &= \frac{1}{x} \\ (x-1)' &= 1 \end{aligned}\right]$$