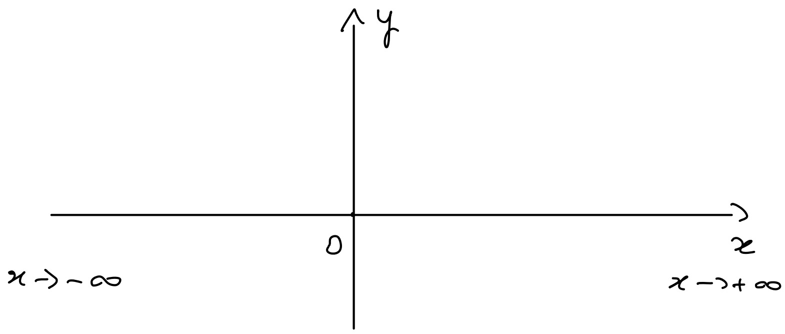
$f : D \subseteq \mathbb{R} \to \mathbb{R}$, D dominio naturale



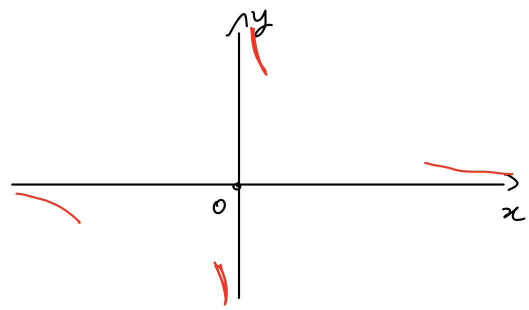
Punti di accumulazione di D

<u>ES</u> $f(x) = \frac{1}{x}$, $D = \{x \in \mathbb{R} : x \neq 0\} = (-\infty, 0) \cup (0, +\infty)$

Punti di acc di D $= \mathbb{R} \cup \{+\infty, -\infty\}$

$\lim_{x \to +\infty} f(x)$, $\lim_{x \to -\infty} f(x)$, $\lim_{x \to 0} f(x)$

$\lim_{x \to +\infty} \frac{1}{x} = 0^+ = \frac{1}{+\infty}$, $\lim_{x \to -\infty} \frac{1}{x} = \frac{1}{-\infty} = 0^-$



<u>OSS</u> $x_0 \in \mathbb{R}$, $\lim_{x \to x_0} f(x)$. Facciamo prima

limite destro $\lim_{x \to x_0^+} f(x) = l \iff \forall\, U$ intorno di l $\exists\, \delta > 0$ t.c.

$\forall x \in (x_0, x_0 + \delta) \cap D$ si ha $f(x) \in U$

limite sinistro $\lim_{x \to x_0^-} f(x) = l \iff \forall\, U$ intorno di l $\exists\, \delta > 0$ t.c.

$\forall x \in (x_0 - \delta, x_0) \cap D$ si ha $f(x) \in U$

<u>Prop</u> Il $\lim_{x \to x_0} f(x)$ esiste ed è uguale a $l \in \mathbb{R} \cup \{+\infty, -\infty\}$ se e solo se i limiti destro e sinistro esistono e sono uguali a l.

$\nexists \lim_{x \to 0} \frac{1}{x}$? <u>NO</u> perché limite dx $\neq$ limite sin.

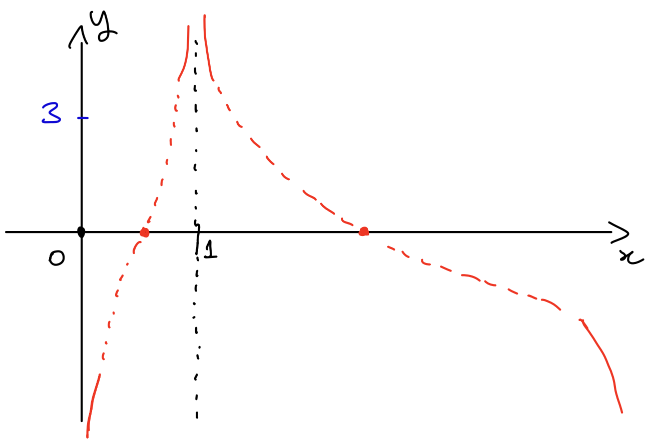
$$\lim_{x \to 0^+} \frac{1}{x} = \frac{1}{0^+} = +\infty \quad , \quad \lim_{x \to 0^-} \frac{1}{x} = \frac{1}{0^-} = -\infty$$

<u>ES</u> $f(x) = \log\left(\frac{x}{(x-1)^2}\right)$

$$D = \Big\{ x \in \mathbb{R} : \underbrace{\frac{x}{(x-1)^2} > 0}_{\text{esponente del log}} \;,\; (x-1)^2 \neq 0 \Big\} = (0,1) \cup (1,+\infty)$$

$$\frac{x}{(x-1)^2} > 0 \iff \begin{cases} (x-1)^2 \neq 0 \\ x > 0 \\ (x-1)^2 > 0 \end{cases} \cup \begin{cases} (x-1)^2 \neq 0 \\ x < 0 \\ (x-1)^2 < 0 \end{cases} \iff \begin{cases} x \neq 1 \\ x > 0 \\ x-1 \neq 0 \end{cases} \cup \begin{cases} x \neq 1 \\ x < 0 \\ \emptyset \end{cases}$$

$$(0,1) \cup (1+\infty) \quad \cup \quad \emptyset$$

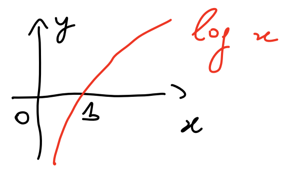


$$\lim_{x \to 0^+} f(x) = -\infty$$

$$\lim_{x \to +\infty} f(x) = -\infty$$

$$\lim_{x \to 1^+} f(x) = +\infty$$

$$\lim_{x \to 1^-} f(x) = +\infty$$



$$\lim_{x \to 0^+} \log\left(\frac{x}{(x-1)^2}\right) = \log(0^+) = -\infty$$

$$\lim_{x \to 0^+} \frac{x}{(x-1)^2} = \frac{0^+}{1} = 0^+$$

$$\lim_{x \to +\infty} \log\left(\frac{x}{(x-1)^2}\right) = \log(0^+) = -\infty$$

$$\lim_{x \to +\infty} \frac{x}{(x-1)^2} = \left(\frac{+\infty}{+\infty} \; f.i.\right) = 0^+$$

$$\frac{x}{(x-1)^2} = \frac{x}{x^2 - 2x + 1} = \frac{x}{x^2\left(1 - \frac{2}{x} + \frac{1}{x^2}\right)} = \frac{\cancel{x}}{x^{\cancel{2}}} \cdot \frac{1}{1 - \frac{2}{x} + \frac{1}{x^2}} =$$

$$= \underbrace{\frac{1}{x}}_{\substack{x\to+\infty \\ \parallel \\ 0^+}} \cdot \underbrace{\frac{1}{1-\frac{2}{x}+\frac{1}{x^2}}}_{\substack{\frac{1}{1-0^+ + 0^+}}}^{} \xrightarrow[x\to+\infty]{} 0^+ \cdot \frac{1}{1} = 0^+ \cdot 1 = 0^+$$

$x \to +\infty$

$$\lim_{x\to 1^+} \log \frac{x}{(x-1)^2} = \log(+\infty) = +\infty$$

$$\lim_{x\to 1^+} \frac{x}{(x-1)^2} = \frac{1}{(0^+)^2} = \frac{1}{0^+} = +\infty \quad \Longrightarrow \quad \exists \lim_{x\to 1} \log\frac{x}{(x-1)^2} = +\infty$$

$$\lim_{x\to 1^-} \log \frac{x}{(x-1)^2} = \log(+\infty) = +\infty$$

$$\lim_{x\to 1^-} \frac{x}{(x-1)^2} = \frac{1}{(0^-)^2} = \frac{1}{0^+} = +\infty$$

$$\log\frac{x}{(x-1)^2} = 0 \iff \frac{x}{(x-1)^2} = 1 \iff x = (x-1)^2 \iff x = x^2-2x+1 \iff x^2-3x+1=0$$

$$\iff x = \frac{3\pm\sqrt{9-4}}{2} = \frac{3}{2} \pm \frac{\sqrt{5}}{2}$$

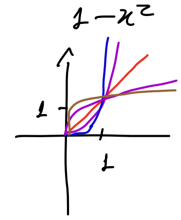
ES $\displaystyle\lim_{x\to 0} \sin\left(\frac{2x^2+3x^4}{x-x^3}\right) = \sin(0) = 0$, $D = \{x\in\mathbb{R} : x-x^3 \neq 0\} = \mathbb{R} \setminus \{0,+1,-1\}$

$$x - x^3 = 0 \iff x(1-x^2) = 0$$

$$\iff x = 0 \ \cup \ x^2 = 1$$

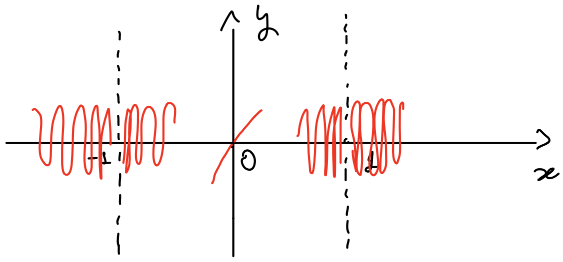
$$\iff x = 0 \ \cup \ x = +1 \ \cup \ x = -1$$

$$\lim_{x\to 0^+} \frac{2x^2+3x^4}{x-x^3} = \left(\frac{0}{0}\ f.i.\right) = \lim_{x\to 0^+} \frac{x^2(2+3x^2)}{x(1-x^2)} = \lim_{x\to 0^+} \frac{x^{\cancel{2}}}{\cancel{x}} \cdot \frac{2+3x^2}{1-x^2} =$$



$$= \lim_{x\to 0^+} x \cdot \frac{2+3x^2}{1-x^2} = 0^+ \cdot \frac{2+0^+}{1-0^+} = 0^+ \cdot 2 = 0^+$$

$$\lim_{x\to 0^-} \frac{2x^2+3x^4}{x-x^3} = \lim_{x\to 0^-} x \cdot \frac{2+3x^2}{1-x^2} = 0^- \cdot \frac{2+0^+}{1-0^+} = 0^- \cdot 2 = 0^-$$



$$\lim_{x \to 1^{\pm}} \frac{2x^2+3x^4}{x-x^3} = \lim_{x \to 1^{\pm}} \frac{2x^2+3x^4}{x(1-x^2)} = \begin{cases} \frac{5}{1 \cdot 0^-} = -\infty & x \to 1^+ \\ \frac{5}{1 \cdot 0^+} = +\infty & x \to 1^- \end{cases}$$

<u>OSS</u> $\frac{e^x}{x^n} \xrightarrow[x \to +\infty]{} +\infty$, $\frac{x^n}{(\log x)^k} \xrightarrow[x \to +\infty]{} +\infty$, $n \geq 1$

$\frac{x^n}{e^x} \xrightarrow[x \to +\infty]{} 0$, $\frac{\log x}{x^n} \xrightarrow[x \to +\infty]{} 0$

$x \to 0^+$ $\quad x^2 \cdot \log x = \frac{\log x}{1/x^2} = \frac{\log \frac{1}{y}}{y^2} = -\frac{\log y}{y^2} \longrightarrow 0$

$0^+ \cdot (-\infty)$ f.i $\quad \frac{-\infty}{+\infty}$ f.i $\quad y = \frac{1}{x} \xrightarrow[x \to 0^+]{} +\infty$

$x \to +\infty$ $\quad x^3 e^{-x} = \frac{x^3}{e^x} \longrightarrow 0$

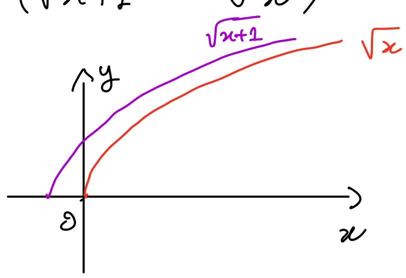
$+\infty \cdot 0^+$

<u>ES</u> $\lim_{x \to +\infty} (x^3 - x^2) = \lim_{x \to +\infty} x^3 \left(1 - \frac{1}{x}\right) = +\infty$

$+\infty - \infty \qquad +\infty \cdot (1-0)$

$+\infty - \infty$

$$\lim_{x \to +\infty} \left(\sqrt{x+1} - \sqrt{x}\right) = \lim_{x \to +\infty} \left(\sqrt{x+1} - \sqrt{x}\right) \frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}} =$$



$$= \lim_{x \to +\infty} \frac{(x+1)-x}{\sqrt{x+1}+\sqrt{x}} = \lim_{x \to +\infty} \frac{1}{\sqrt{x+1}+\sqrt{x}} = \frac{1}{+\infty} = 0^+$$

Limiti notevoli

- $\lim_{x\to 0} \frac{\sin x}{x} = 1$, $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$, $\lim_{x\to 0} \frac{\tan x}{x} = 1$

  $\frac{0}{0}$ f.i. $\qquad$ $\frac{0}{0}$ f.i

- $\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$

  $\frac{0}{0}$

ES $\lim_{x\to 0} \frac{x \sin x}{1-\cos x} = 2$

$$\frac{x \sin x}{1-\cos x} = x \cdot \underbrace{\left(\frac{\sin x}{x}\right)}_{\to 1} x \cdot \frac{1}{x^2} \underbrace{\left(\frac{x^2}{1-\cos x}\right)}_{\to 2}$$